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Crystallography. — On M. BEREK's *Methode der Charakteristischen Gangunterschiedsverhältnisse*. By W. NIEUWENKAMP. (Communicated by Prof. L. RUTTEN).

(Communicated at the meeting of January 29, 1927).

In his excellent work „Mikroskopische Mineralbestimmung mit Hilfe der Universaldrehtischmethoden” M. BEREK points out a new method for determining the optic axial angle, the optical character and the position of the optic axes of symmetry of a mineral, requiring only the measurement of some six or seven values of wave differences by means of the compensator after BEREK. These measurements are simple and convenient, but BEREK derives from them the axial angle by means of rather long and tedious calculations. I hope I have shown in this paper that these calculations can be simplified without diminishing the accuracy, and that in this way I have made this really fine method more efficient.

For a clear understanding it is desirable to describe first shortly the instrumental part of the method: The slide is clamped on a Federow-stage, between two hemispheres of glass, whose refractive index is as much as possible like that of the mineral under test, and we place in the ordinary way an optic plane of symmetry perpendicular to the axis A_4 , i.e. the second (always) horizontal axis of rotation. Then the whole stage is rotated about the axis A_5 , (the axis of the microscope) for 45° , so that the direction of the vibration of the fast ray in the mineral and in the compensator are at right angles to one another, and we can start our measurements. They are taken in a series of different directions (all of them in the plane of symmetry) by placing the mineral by rotation about A_4 in different positions, e.g. successively on the readings: $\alpha_4 = -50^\circ, -40^\circ, -20^\circ, 0^\circ, 20^\circ, 40^\circ, 50^\circ$. In this way we get a series of readings at the compensator, and in the table *Kompensatorfunktion* 10 000 $f(i)$, (p. 164 *Drehtischmethoden*) we can find a series of values proportional with the wave differences. They need not be multiplied with the constant of the compensator, for their rationes will be the same as those of the actual wave differences, and, as is suggested by its name, this method makes only use of these rationes. When we wish to intercompare the values of $f(i)$, they must first be reduced to the same length of way of the light in the mineral.

BEREK reduces them to the real thickness of the slide. It is unnecessary to be so scrupulous; the only condition is that the length of the way of the light has to be the same for every observation. The reduction becomes very simple when we reduce the observation to the interval

the ray passes when A_4 stands in its initial position ($\alpha_4 = 0^\circ$). This is shown in fig. 1.

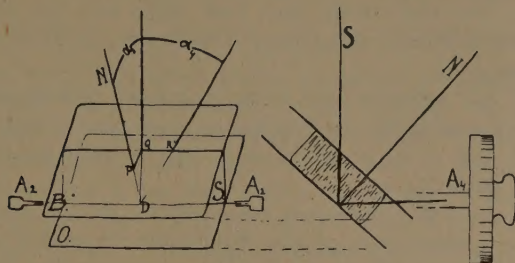


Fig. 1.

Here O , and B represent the under and upper surface of the slide, N is drawn perpendicular to it. S is the plane of symmetry in which we measure the wave differences; so the ray passes always through the mineral along one of the lines of S ; when A_4 stands in its initial position ($\alpha_4 = 0^\circ$) along the line DQ , when the stage is inclined, along a line DR .

In the initial position the axis A_2 is horizontal. Consequently in that position also the line QR , i.e. the trace of the plane of symmetry runs horizontal, for our first manipulation was placing this plane parallel to A_2 . The ray, in this case DQ , is vertical, so DQ is at right angles with QR , and $QD:RD = \cos \alpha_4$; therefore in order to reduce the way travelled over by the light to DQ we have only to multiply by $\cos \alpha_4$.

In BEREK's procedure the reduction runs as follows: he begins by plotting a curve with the measured wave differences as ordinates, and α_4 as abscissae. On the same abscissae he also plots the ordinates that represent the angle RDQ , corresponding with every α_4 , i.e. the angle between the ray and the normal on the slide. In a stereographic projection in which the plane of symmetry and the ray for every α_4 are represented, these angles are measured by means of a WULLF's net, and so we can multiply the ordinates of every point of the first curve by the $\cos$ of the corresponding angle RDQ thus reducing indeed, all the wave differences to the real thickness of the slide.

In this way then BEREK obtains the reduced wave difference-curve. In this curve a maximum or a minimum has to be looked for; it is however not easy to find where it lies exactly. According to BEREK one begins by assuming a presumptive value for the abscissa of the maximum or minimum, and by marking out distances from it of 30° , 45° and if possible 60° , 75° and 90° , then the corresponding ordinates are divided by the ordinates of the maximum or minimum, and, we try to find in 5 diagrams, constructed by BEREK, and added to his work, by means of these 5 quotients 5 values for the axial angle. If these values are not equal, if their curve runs up or falls then the choice of the abscissa of the minimum is wrong and we have to do it over again. If now their

curve moves in the other direction, we can find by means of interpolations and by other trials an abscissa for which the axial angle is the same for all quotients; this then is the real axial angle and the value of the abscissa of the maximum or minimum indicates the position of an optical axis of symmetry.

I think I can substitute the latter operation of trial and error by moving a curve projected on transparent paper, and obtained from the observations, over a set of curves on a diagram until the best possible agreement is attained.

That this can really be done is borne out by the following consideration: in our computation of the axial angle we availed ourselves of rationes of the wave differences; it is these rationes that determine the axial angle. When plotting the logarithms of the wave differences the shape of the curve will depend on the axial angle. The lengths of the ordinates representing the logarithms of the wave differences indeed also depend on the thickness of the slide and the compensator constant, but the shape of the curve is determined by the differences of the ordinates, and these differences represent the logarithms of the quotients of the wave differences and consequently depend only on the axial angle. So with each axial angle there is a definite shape of the curve of the log. wave differences. Now the later have been drawn on the diagrams for axial angles varying from 0° — 90° . The absolute lengths of the ordinates are chosen so as to be most convenient for the construction and the selection of a definite curve.

From our theoretical discussions at the close of this paper it will appear that 2 diagrams are required, one for the optic axial plane and one for the 2 planes of symmetry perpendicular to it, but first we shall illustrate the use of the diagrams by a few examples.

1	2	3	4	5	6
α_4	Wave differences	$i = \frac{a-b}{2}$	log wave differences	$\log \cos \alpha_4$	Reduced log wave differences
50	193.5	25.55	9.287	0.809—1	9.096
40	183.0	24.83	9.263	0.884—1	9.147
20	176.1	24.35	9.246	0.973—1	9.219
0	173.0	24.12	9.238		9.238
—20	170.6	23.95	9.232	0.973—1	9.205
—40	168.9	23.83	9.228	0.884—1	9.112
—50	175.2	24.28	9.243	0.809—1	9.052

In BERK's work we read on p. 121 that when measuring a plagio-klase he found corresponding with the α_4 from column 1 the wave

differences as indicated in column 2. These are not the real wave differences but the values (proportional with the wave differences) of table:

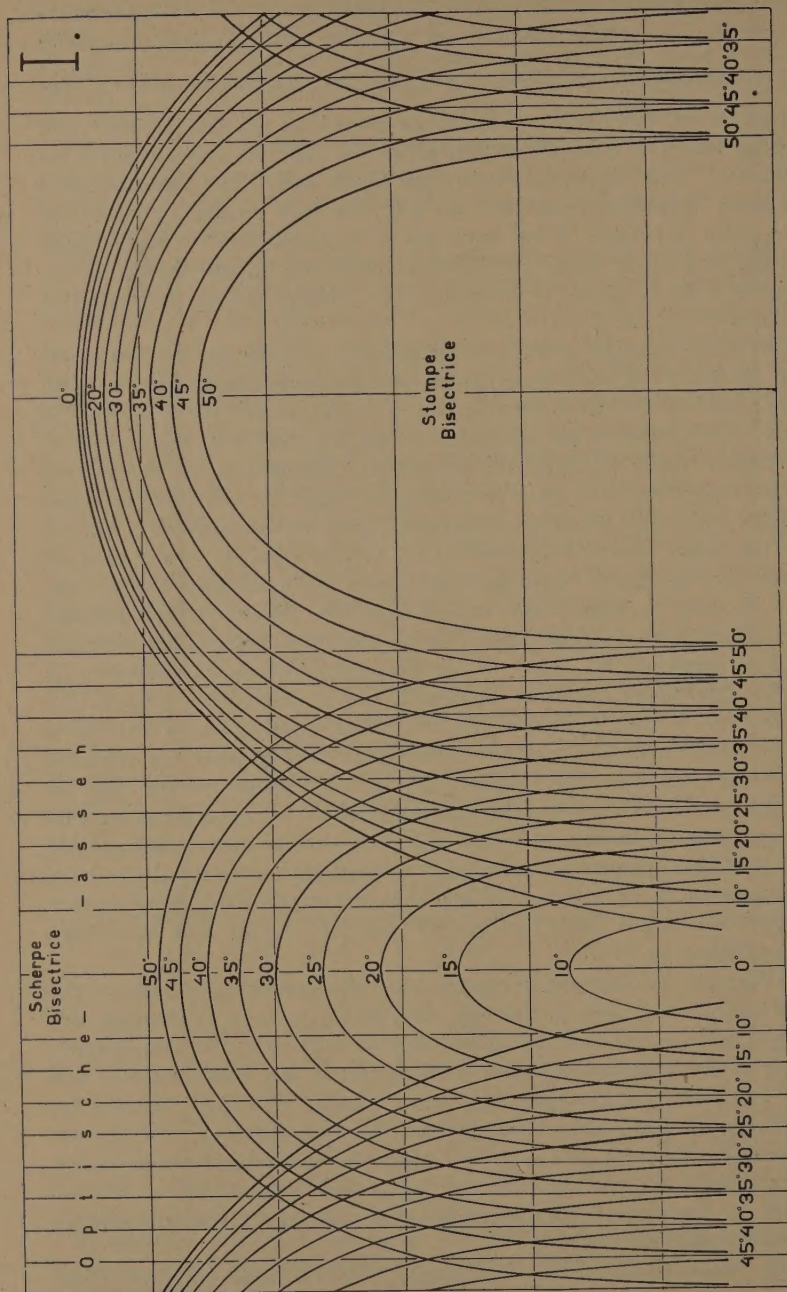
"Kompensatorfunktion 10 000 $f(i)$ ", and we can find the $i = \frac{a-b}{2}$ (not given in his work) with which BEREK started his calculations, and upon which we will base our example. So we take the i of column 3 for measured with the compensator, and in the table "Für logarithmisches rechnen, Kompensatorfunktion $\log f(i)$ ", we look up the $\log f(i)$. They are given in column 4, but have still to be reduced to the same length of the way of the light. Therefore we have only to add the $\log \cos a_4$. $\log \cos a_4$ is given in column 5, by addition we get the values of column 6.

Now the calculations are already finished, and we can plot the curve. To get it on the same scale as the accompanying diagrams we have to mark off on the axis of abscissae 1 cm for every 10° of a_4 ; on the axis of ordinates 5 cm for every unit of the logarithm. On a sheet of millimeter paper this scale for abscissa and ordinate should be marked; if we cover it with a sheet of transparent paper the dotting of the curve on the right scale is an easy operation. Care should be taken to indicate at the same time the direction of the axis of abscissae, which may be done by drawing a horizontal line across the transparent paper. Care should then be taken that while moving the paper over the diagram, we always keep this line parallel to the horizontal lines of the diagram.

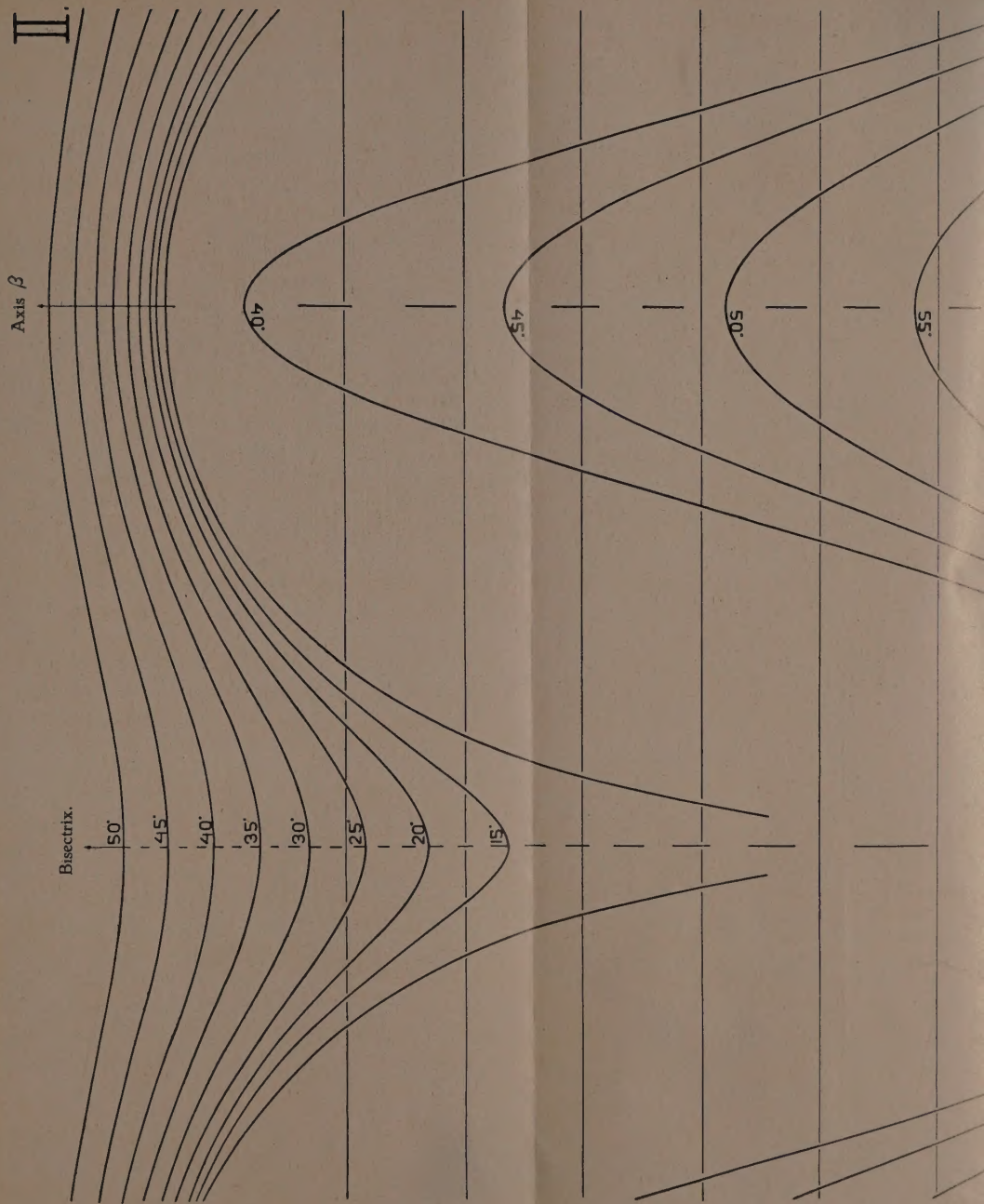
The shape of the curve of our example shows directly that it must fit to one of the curves of diagram 2, and is intermediate between the curve for $V 40^\circ$, and that for 45° . If the curve bends so little as in the present case we may for convenience' sake magnify the ordinates 10 times and then use diagram 2^A, a magnification of the top part of diagram 2. With both diagrams the result is of course the same. The correspondence to the curve for $42\frac{1}{2}^\circ$ is not yet quite sufficient. We see that our points are lying in a curve slightly less arched, we have therefore to go up a little, and shall then find that the best correspondence is obtained with a curve between 43° and 44° . So the total axial angle is established at 87° .

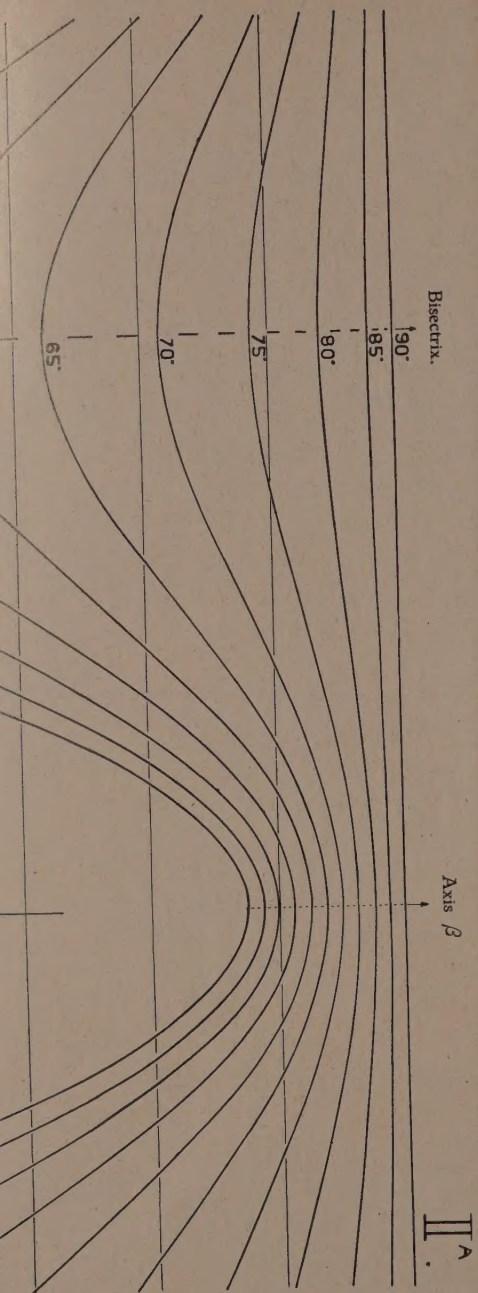
When in plotting the curve we have also indicated the scale of the abscissa we are able to indicate directly the position of an optic axis of symmetry, (in this case the optical normal, with $a_4 = 3^\circ$). The bisectrix lying in the contemplated plan of symmetry, is that which has the angle V , indicated in the diagram, on either side, so it is acute bisectrix for the curves 0° — 45° , and obtuse bisectrix for those from 45° — 90° (in our case acute bisectrix). We now also know the third axis of symmetry and consequently we can also indicate the optical character of the mineral, when we know from our compensator the directions of the vibration of the slow and fast ray.

An example of the use of diagram 1 is to be found on page 116 of



W. NIEUWENKAMP: ON M. BEREK'S "METHODE DER CHARAKTERISTISCHEN
GANGUNTERSCHIEDSVERHÄLTNISSE".





the "Drehtischmethoden". This is a measurement made in the optic axial plane. We have not taken measures close to the optical axis, because there the small wave differences render them too inaccurate; on the other hand we have a determination of the abscissa of the optical axis; on the transparent sheet it is marked by a vertical line with $\alpha_4 = 21,2^\circ$, that is the α_4 with which BEREK has found the optical axis. For the other points of the curve we find in the same way as in the first example from BEREK's wave differences:

$\alpha_4 =$	50°	40°	10°	0°	-20°	-40°	-50°
log wave diff. =	8,706	8,618	8,529	8,811	9,057	9,105	9,077

It follows from the fact that the measurements have been made in the plane of the axes, that diagram I is to be used, which moreover is shown clearly by the shape of the curve. When fitting the curve on the diagram care should be taken that the same angle V is found on either branch of the curve and also that the same V stands at the line marking the position of the optical axis. We can place the right part of our curve first on the curve for 35° , but then the left branch lies at 32° (anyhow if we have laid the line for the optical axis along that for 35°). So we have to shove the paper upward slantingly to the left. With 33° there is a fair concordance, with 32° it decreases already. So we find $V = 32\frac{1}{2}^\circ$ ($2V = 65^\circ$), in complete accordance with BEREK's result.

Here also we can easily mark where in our mineral an optic axis of symmetry is to be found, and which of the axes it is, as well as the remaining 2 axes and the optical character.

One more remark about this graphical arithmetical method:

The points dotted on the transparent paper have not been connected by a continuous line; this is quite irrelevant as we do something like it when shoving the paper over the diagrams. Nay, it is better even to let the points alone, because otherwise we should most likely draw a curve of a shape that cannot occur with perfectly correct measurements, and is impossible with any axial angle, whereas in the shoving process the points are connected by a curve that is theoretically possible. In this case therefore we make a selection from curves that of themselves may occur, though perhaps with another axial angle, whereas in the first case we can hit upon a good curve only with perfectly correct measurements. Generally however we shall find one that will give trouble rather than pleasure.

The data for the construction of the diagram are found in the following way: According to BEREK (p. 154 sqq. of his Drehtischmethoden), when our measurements are taken in an optical plane of symmetry which is not the axial plane:

$$\frac{\sin(r, n_x)}{\sqrt{\frac{\Delta_r}{\Delta_x} - 1}} = \operatorname{tg} V \quad \text{or} \quad \frac{\sin(r, n_x)}{\sqrt{1 - \frac{\Delta_r}{\Delta_x}}} = \sec V.$$

Here r is the direction of the ray, n_x an optical axis of symmetry, Δ_r , Δ_x the wave differences in the direction r resp. n_x . If furthermore V is counted from 0° — 90° , one trigonometrical function will do (it is then needless to mention: or $\cotg V$, resp. $\operatorname{cosec} V$).

From the first formula follows:

$$\frac{\Delta_r}{\Delta_x} = \frac{\sin^2(r, n_x)}{\operatorname{tg}^2 V} + 1 = \frac{\sin^2(r, n_x) + \operatorname{tg}^2 V}{\operatorname{tg}^2 V}.$$

$$\lg \Delta_r = \lg \{ \sin^2(r, n_x) + \operatorname{tg}^2 V \} - \lg \operatorname{tg}^2 V + \lg \Delta_x.$$

FOR DIAGRAM I. $\lg |\sin^2(r, n_x) - \sin^2 V|$

$\begin{matrix} V \\ (r, n_x) \end{matrix}$	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
0°		7.881	8.479	8.826	9.068	9.252	9.398	9.517	9.616	9.699
5°	7.881		8.352	8.774	9.039	9.233	9.384	9.507	9.608	9.692
10°	8.479	8.352		8.567	8.939	9.172	9.342	9.476	9.583	9.672
15°	8.826	8.774	8.567		8.699	9.048	9.262	9.418	9.539	9.636
20°	9.068	9.039	8.939	8.699		8.790	9.123	9.326	9.472	9.583
25°	9.252	9.233	9.172	9.048	8.790		8.854	9.177	9.370	9.507
30°	9.398	9.384	9.342	9.262	9.123	8.854		8.898	9.213	9.398
35°	9.517	9.507	9.476	9.418	9.326	9.177	8.898		8.925	9.233
40°	9.616	9.608	9.583	9.539	9.472	9.370	9.213	8.925		8.938
45°	9.699	9.692	9.672	9.636	9.583	9.507	9.398	9.233	8.938	
50°	9.768	9.763	9.746	9.716	9.672	9.611	9.527	9.411	9.239	8.938
55°	9.827	9.822	9.807	9.781	9.744	9.692	9.624	9.534	9.411	9.233
60°	9.875	9.871	9.857	9.834	9.801	9.757	9.699	9.624	9.527	9.398
65°	9.915	9.910	9.898	9.878	9.848	9.808	9.757	9.692	9.611	9.507
70°	9.946	9.942	9.931	9.912	9.884	9.848	9.801	9.743	9.672	9.583
75°	9.970	9.966	9.956	9.937	9.912	9.878	9.834	9.781	9.716	9.636
80°	9.987	9.983	9.973	9.956	9.931	9.898	9.857	9.807	9.746	9.672
85°	9.997	9.993	9.983	9.966	9.942	9.911	9.871	9.822	9.763	9.692
90°	10	9.997	9.987	9.970	9.946	9.915	9.875	9.827	9.769	9.699

FOR DIAGRAM II, $I_g \{ \sin^2(\tau, n_\tau) + tg^2 V \}$.

$\frac{V}{(\tau, n_\tau)}$	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°
0°		0.8839—3.0	0.4926—2.0	0.8561—2.0	0.1221—1.0	0.3373—1.0	0.5230—1.0	0.6905—1.0	0.8476—1.0	0.1524	0.3095	0.4771	0.6627	0.8779	1.1439	1.5075	2.1159	
5°	0.8806—3.0	0.1833—2.0	0.5876—2.0	0.8998—2.0	0.1464—1.0	0.3523—1.0	0.5327—1.0	0.6971—1.0	0.8523—1.0	0.0033	0.1547	0.3111	0.4782	0.6635	0.8784	1.1442	1.5076	2.1160
10°	0.4793—2.0	0.5776—2.0	0.7878—2.0	0.0084—1.0	0.2111—1.0	0.3937—1.0	0.5595—1.0	0.7164—1.0	0.8658—1.0	0.0129	0.1615	0.3159	0.4815	0.6656	0.8797	1.1449	1.5079	2.1161
15°	0.8260—2.0	0.8730—2.0	0.9917—2.0	0.1423—1.0	0.2998—1.0	0.4540—1.0	0.6025—1.0	0.7461—1.0	0.8870—1.0	0.0282	0.1724	0.3235	0.4867	0.6690	0.8818	1.1460	1.5084	2.1162
20°	0.0681—1.0	0.0956—1.0	0.1706—1.0	0.2759—1.0	0.3970—1.0	0.5243—1.0	0.6535—1.0	0.7834—1.0	0.9144—1.0	0.0481	0.1867	0.3337	0.4937	0.6737	0.8846	1.1476	1.5090	2.1163
25°	0.2519—1.0	0.2701—1.0	0.3216—1.0	0.3986—1.0	0.4929—1.0	0.5977—1.0	0.7092—1.0	0.8254—1.0	0.9458—1.0	0.0714	0.2038	0.3460	0.5022	0.6793	0.8881	1.1495	1.5098	2.1165
30°	0.3979—1.0	0.4110—1.0	0.4489—1.0	0.5076—1.0	0.5826—1.0	0.6697—1.0	0.7659—1.0	0.8694—1.0	0.9796—1.0	0.0969	0.2228	0.3597	0.5119	0.6857	0.8921	1.1517	1.5108	2.1167
35°	0.5172—1.0	0.5272—1.0	0.5564—1.0	0.6029—1.0	0.6642—1.0	0.7375—1.0	0.8211—1.0	0.9134—1.0	0.0141	0.1235	0.2429	0.3745	0.5223	0.6928	0.8965	1.1541	1.5119	2.1170
40°	0.6161—1.0	0.6241—1.0	0.6477—1.0	0.6857—1.0	0.7370—1.0	0.7997—1.0	0.8730—1.0	0.9559—1.0	0.0482	0.1502	0.2633	0.3896	0.5332	0.7001	0.9011	1.1566	1.5130	2.1173
45°	0.6990—1.0	0.7056—1.0	0.7252—1.0	0.7572—1.0	0.8011—1.0	0.8558—1.0	0.9208—1.0	0.9958—1.0	0.0807	0.1761	0.2834	0.4047	0.5441	0.7076	0.9058	1.1593	1.5142	2.1176
50°	0.7685—1.0	0.7741—1.0	0.7909—1.0	0.8186—1.0	0.8569—1.0	0.9054—1.0	0.9638—1.0	0.0323	0.1109	0.2005	0.3025	0.4193	0.5547	0.7149	0.9105	1.1619	1.5153	2.1179
55°	0.8267—1.0	0.8317—1.0	0.8465—1.0	0.8709—1.0	0.9050—1.0	0.9487—1.0	0.0019	0.0650	0.1384	0.2230	0.3204	0.4330	0.5648	0.7219	0.9149	1.1644	1.5164	2.1182
60°	0.8751—1.0	0.8794—1.0	0.8927—1.0	0.9148—1.0	0.9457—1.0	0.9856—1.0	0.0348	0.0935	0.1626	0.2430	0.3365	0.4455	0.5740	0.7284	0.9191	1.1667	1.5174	2.1184
65°	0.9146—1.0	0.9186—1.0	0.9307—1.0	0.9509—1.0	0.9795—1.0	0.0165	0.0625	0.1178	0.1834	0.2604	0.3506	0.4565	0.5822	0.7341	0.9228	1.1688	1.5184	2.1187
70°	0.9460—1.0	0.9497—1.0	0.9610—1.0	0.9800—1.0	0.0067	0.0416	0.0851	0.1378	0.2006	0.2749	0.3623	0.4658	0.5892	0.7390	0.9260	1.1706	1.5192	2.1189
75°	0.9699—1.0	0.9734—1.0	0.9841—1.0	0.0021	0.0276	0.0609	0.1025	0.1533	0.2141	0.2862	0.3717	0.4731	0.5947	0.7430	0.9285	1.1721	1.5199	2.1190
80°	0.9867—1.0	0.9901—1.0	0.0004	0.0177	0.0423	0.0745	0.1150	0.1644	0.2237	0.2944	0.3784	0.4785	0.5988	0.7458	0.9304	1.1732	1.5203	2.1191
85°	0.9967—1.0	0.0000	0.0101	0.0270	0.0511	0.0827	0.1225	0.1711	0.2295	0.2994	0.3825	0.4817	0.6012	0.7476	0.9316	1.1738	1.5206	2.1192
90°	0.	0.0033	0.0133	0.0301	0.0540	0.0854	0.1249	0.1733	0.2315	0.3010	0.3839	0.4828	0.6021	0.7482	0.9320	1.1741	1.5207	2.1193

From the second formula follows:

$$\frac{\Delta_r}{\Delta_x} = \frac{-\sin^2(r, n_x)}{\sec^2 V} + 1 = \frac{\cos^2(r, n_x) - 1 + (1 + \operatorname{tg}^2 V)}{1 + \operatorname{tg}^2 V},$$

$$\lg \Delta_r = \lg [\sin^2 \{ (r, n_x) \pm 90^\circ \} \operatorname{tg}^2 V] - \lg (1 + \operatorname{tg}^2 V) + \lg \Delta_x.$$

The shape of the curve for a definite V is now in the first case the same as in the second, so one curve suffices for both cases, if only in the second case we begin to measure the angle (r, n_x) from a point that differs 90° from the n_x in the first case.

In the axial plane (see BEREK l. c.) is:

$$\frac{\sin^2(r, n_x)}{1 \pm \frac{\Delta_r}{\Delta_x}} = \sin V, \quad (V \text{ counted from } 0^\circ - 90^\circ) \text{ from which follows:}$$

$$\frac{\Delta_r}{\Delta_x} = \left| \frac{\sin^2(r, n_x)}{\sin^2 V} - 1 \right| = \frac{|\sin^2(r, n_x) - \sin^2 V|}{\sin^2 V}$$

$$\lg \Delta_r = \lg |\sin^2(r, n_x) - \sin^2 V| - \lg \sin^2 V + \lg \Delta_x.$$

For the construction of the diagrams, which have to indicate for each definite V , the differences between $\lg \Delta_r$ when the angle (r, n_x) varies, only that part of the formula is of consequence in which (r, n_x) occurs; it does not matter if a constant is added or not. So for the construction we require the formulae:

$$\lg \Delta_r = \lg \{ \sin^2(r, n_x) + \operatorname{tg}^2 V \} \quad \text{resp.} \quad \lg \Delta_r = \lg |\sin^2(r, n_x) - \sin^2 V|$$

For those who should wish to attain a greater accuracy than is possible with the small diagrams reproduced here, and would construct them on a larger scale, the results have been tabulated here.

That as will be seen in formula 2, the $\lg \Delta_r$ becomes $-\infty$ for $(r, n_x) = V$, need not discompose us very much, because (on BEREK's suggestion) we do not take any measurements in the neighbourhood of an optical axis, so that there we have no points to be fitted on the curve, and consequently a part of it can be missing without vitiating the results.

Comparative Pathology. — *On the mutual Immunization in Aphthae epizooticae and Vaccine.* (First very concise memoir). By Prof. C. F. VAN OIJEN. (Communicated by Prof. C. EIJKMAN).

(Communicated at the meeting of March 26, 1927).

It has been recorded repeatedly for more than a quarter of a century in the veterinary literature, that animals previously treated with "vaccine", are in a measure immune from Foot-and-mouth disease.

BOULLAND ¹⁾ (1900) reports that in his private practise he observed that cows which had suffered from the genuine cowpox ("variolaë"), were not taken with foot-and-mouth disease, when this broke out in the same stable.

SEIBERT ²⁾ (1907) stated that a calf which had served for the preparation of "vaccine" was not affected, when the other cows of the stable in which it was brought afterwards, were seriously afflicted with aphthea. He translated these observations into actions by inoculating calves with 20—25 pocks. After this the calves are said to have been immune against foot-and-mouth disease.

ORY ³⁾ (1907) inoculated cows with lymph, derived from horses, which had been inoculated with "vaccine", and observed, that several of these animals did not get ill, when they were seriously infested with substances containing aphthae-virus (saliva etc.).

ANKER ⁴⁾ (1907) (Oudewater) could not fully confirm ORY's findings. Only a group of young animals ($1\frac{1}{2}$ years) being inoculated with pocks and subsequently exposed to infection with aphthae, remained free from this disease.

Neither could UHLENHUTH and BIEBER ⁵⁾ establish the immunity in question for all cases, although the critical reader will observe that it did exist in some vaccinated animals.

For a complete survey of the relative experiments we refer to GINS

¹⁾ BOULLAND. Le cow-pox et la fièvre aphteuse. Le répertoire de police sanitaire vétérinaire et d'hygiène publique. Vol. 16 1900. 15 Sept. N^o. 9, p. 377.

²⁾ SEIBERT. Kuhpocken Impfung als Schützmittel gegen Maul- und Klauenseuche. Wochenschrift für Tierheilkunde und Viehzucht. 1 Oct. 1907 N^o. 39.

³⁾ ORY. Essai d'un mode prophylactique de la fièvre aphteuse par la vaccination. La Semaine Vétérinaire 1907 p. 279.

⁴⁾ ANKER. Maul- und Klauenseuche Impfung nach ORY. Berliner Tierärztliche Wochenschrift 1907 p. 882, 883.

⁵⁾ UHLENHUTH und BIEBER. Untersuchungen zur Frage der wechselseitigen Vakzine- und Maul- und Klauenseuche Immunität bei Rindern und Meerschweinchen. Zeitschrift für Immunitätsforschung. Originale Bnd. 35 1923 Heft 4 p. 10.

and KRAUSE¹⁾: "Zur Pathologie der Maul- und Klauenseuche sub. IX Pocken und Maul- und Klauenseuche".

Also the present author was in a position to observe in his private practice that two calves, which had served to prepare "vaccine", did not get ill when housed in the same stable with animals with foot- and -mouth disease.

These observations, based on practice, should not be underrated. Therefore, GINS and KRAUSE and others advocate the urgency of an experimental inquiry into the power of a vaccine-inoculation to protect from foot- and -mouth disease. This is all the more desirable, as in 1926 a report was received from France that very good results were obtained by inoculation of a definite mixture of vaccine and aphthae-virus (Belin).²⁾

We purpose to prosecute this inquiry with cows and calves. But these experiments are expensive, while it is very difficult to avoid technical errors. For this reason we also made experiments with infected caviae. The experiments with cows will be published afterwards.

However, the cavia-experiments have reached a stage, at which they can for the present be concluded. A brief account of these experiments follows:

Detailed reports with protocols etc. will soon be published.

The investigation was divided as follows:

- I. Description of the symptoms of foot- and -mouth disease in caviae.
- II. Idem of the symptoms of vaccine ("cowpox") in caviae.
- III. Observations regarding the immunity of caviae from vaccine after inoculation with this virus.
- IV. Observations on the immunity from aphthae-virus after inoculation with this infectious matter.
- V. Observations on the immunity of caviae from foot- and -mouth disease after inoculation with vaccine.
- VI. Observations on the immunity of caviae from vaccine after inoculation with aphthae virus.

Graphs have been plotted for vaccine- as well as for aphthae-infections, in which the succession of the symptoms, their intensity and their duration may be read at a glance. The line *AP* indicates the changes at the hindlegs, *VP* those at the forelegs, *T* at the tongue.

I. Description of the *symptoms* of *foot- and mouth disease* in caviae.

We observed:

- 1⁰. The body-temperature. After 1 to 2 days' incubation the temperature

¹⁾ GINS und KRAUSE. Zur Pathologie der Maul- und Klauenseuche Ergebnisse der Allgemeinen Pathologie und Patholog. Anatomie des Menschen und der Tiere 1924 vol. II Abt. II.

²⁾ BELIN. Base scientifique d'une méthode de Vaccination antiaphteuse. Recueil de médecine-vétérinaire 1926. Tome 102, N^o. 14 p. 307.

rises in typical cases and is followed by a fall, occasionally below the normal temperatures. It is soon restored to a slightly higher level. For a long time the temperature remains variable (Fig. 1).

2°. The body-weight. In all the cases there is a heavy loss of body-weight (Fig. 1) from which the animals recover but slowly. Generally the initial weight is not regained before the lesions at the site of inoculation are cured.

3°. The development and healing of the apthae-blains at the site of inoculation (sole of the foot metatarsus hindlegs). After the incubation blains appear, then their regression, formation of scabs, shedding of these scales, appearance of a skin-lesion, its healing.

4°. Alterations at the soles of the fore-legs. Here we observe redness, swelling, afterwards exfoliation of the skin and ultimate healing.

5°. Changes of the mucous membrane of the mouth. Blains appear in the mouth, especially on the tongue, that bring on a loss of epithelium and often excoriation all over the tongue. Cure, as a rule, proceeds rapidly.

6°. Some other manifestations, so staring that they, so to speak, obtrude themselves to be taken up in the protocols, such as considerable falling off of the hair, hyper secretions from the salivary glands and of intense salivation.

II. Description of the *symptoms* after inoculation with *vaccine* in *caviae* (Fig. 2).

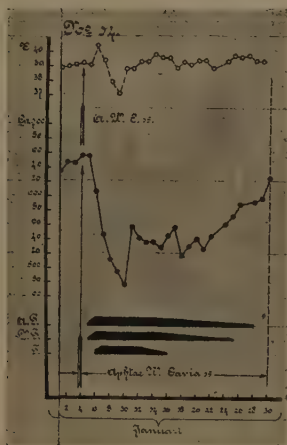


Fig. 1.

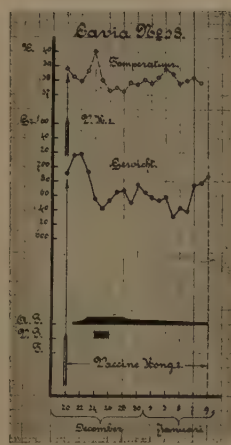


Fig. 2.

We observed:

1°. Changes in the body-temperature. In most cases fever ensues from

3 to 5 days after the inoculation. Gradually a fall succeeds and a varying temperature for some days. A fall below the "normal" temperature was not observed.

2°. Changes in the body-weight. Here also a considerable loss is often seen after the incubation period, truly, not so heavy as with aphthae, but sufficient to discern that the inoculation with vaccine has not the effect of a localized inflammation but influences the general condition of the animal. It makes the animal ill.

3°. Growth of the blain and healing of the area of inoculation. A simple formation of blains exists here, that heals under a scab. After the first scales have fallen off there may appear a wound-surface, which heals in the usual way.

4°. Changes at the soles of the fore-legs. Only redness and swelling are observed here. No other symptoms appeared after the ordinary inoculation with vaccine.

III. Observations regarding the *immunity* of caviae from *vaccine* after inoculation with *this virus*.

A vaccine-injection imparts to a cavia complete immunity against a re-inoculation with this virus, administered immediately after recovery. It does not matter, whether for the re-inoculation the same vaccine-strain is employed or an other. A transit through cavia or calf is of no influence.

From the fact that the symptoms mentioned in II, were totally absent, we inferred the existence of this complete immunity. Fig. 3 illustrates this. It will be seen, that in the control-cavia N°. 26 the symptoms have been

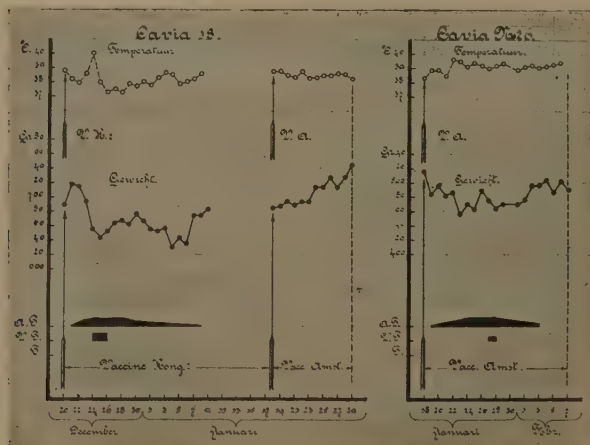


Fig. 3.

fully developed, while in the cavia N^o. 18 no anomalies could be recognized. This immunity could be established in 5 caviae.

IV. *Immunity* in caviae from *aphthae epizooticae* after injection of *this virus*.

A single infection with a very virulent aphthae-strain is sufficient to evoke complete immunity against this virus. This is instanced by cavia N^o. 3 (Fig. 4). The accompanying curve shows distinctly the difference between cavia 3 after re-inoculation and cavia 10 after the first injection with the same infectious matter. The same immunity appears against other strains of equal virulence. Cavia N^o. 21 (control-cavia 43).

Yet there is a large difference regarding the facts found with vaccine.

When a cavia is inoculated with an aphthae-strain that is only slightly virulent for this animal, the symptoms will be confined to formation of blains at the hindlegs, redness at the fore-legs, loss of weight and an irregular temperature (Fig. 5, cavia N^o. 7). At a following inoculation

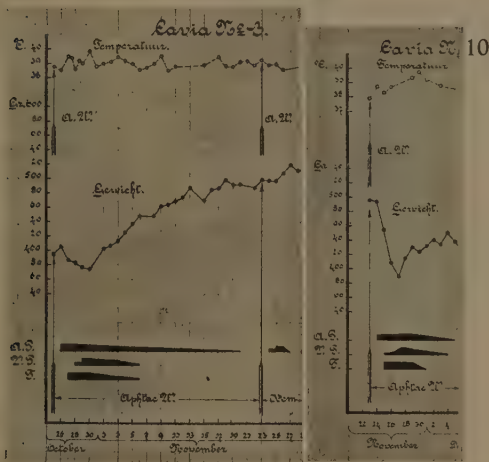


Fig. 4.

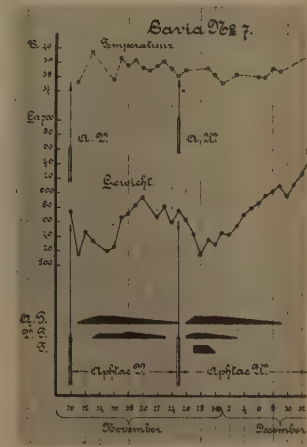


Fig. 5.

with a very virulent strain it appears that no immunity from this virus has been effected. Heavy loss of weight, affections at the hindlegs, forelegs and tongue appear just as with a cavia that has not been treated previously. Only there is a tendency to quicker healing (allergic reaction GINS and WEBER¹⁾).

This remarkable "plurality" of the aphthae-strains could be established in three caviae. Here we wish to refer to the observations made by

¹⁾ GINS und WEBER. Ueber experimentelle Maul- und Klauenseuche. Centralblatt f. Bakteriologie etc. Erste Abt. Originale Bnd. 88, 1922, Heft 3 p. 180.

VALLÉE and CARRÉ¹⁾, WALDMANN and TRAUTWEIN²⁾ and STOCKMANN and MINETT³⁾ in which the same symptoms were recognized.

V. Immunity against aphthae after inoculation with vaccine.

In the foregoing we have dwelt on the nature and the scope of the immunity from a specific virus after inoculation with vaccine and aphthae. Now we could pass on to an inquiry into the mutual immunity with cross-inoculation. From this it appeared that *inoculation with vaccine does not induce immunity against aphthae when administered directly after the healing.*

In fig. 6 we see that a cavia tolerates a vigorous vaccine infection

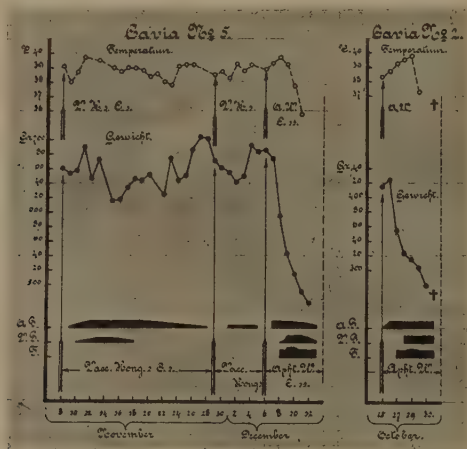


Fig. 6.

with fever, loss of weight, anomalies at the fore-, and hindlegs. Then it appears to be entirely immune against a repeated vaccine-inoculation.

But the subsequent inoculation with aphthae-virus induces death within 6 days. The figure shows that the process is identical with the result obtained with cavia N^o. 2, which had not been treated beforehand. It was also observed in cavia N^o. 1 that a previous inoculation with vaccine acted no influence upon the course of the subsequent inoculation with aphthae, though the latter did not induce death.

¹⁾ VALLÉE H. et CARRÉ H. Sur l'immunité antiaphtheuse Compt. rend. de l'Acad. des Sciences 1922, T. 174, p. 207.

²⁾ WALDMANN und TRAUTWEIN. Experimentelle Untersuchungen über die Pluralität der Maul- und Klauenseuche Virus (Vorl. Mitteil.) Berl. Tierarztl. Wochenschr. Jrg. 42, N^o. 35.

³⁾ ST. STOCKMANN and F. C. MINETT. Experiments on Foot- and Mouth-disease. The journal of Comp. Path. and Therapeutics, Vol. 39, Part 3.

These observations stand in harmony with the findings established by UHLENHUTH and BIEBER¹⁾

VI. It still remains for us to watch the effect of a preceding inoculation with aphthae upon a vaccine-inoculation following it immediately.

In ten caviae it could be established that *sustaining a serious general attack of aphthae epizooticae engenders an almost complete immunity against an infection with vaccine succeeding the attack directly.*

Graphs of cavia 10 and cavia 4 go to illustrate this.

It may be seen that a single inoculation with aphthae into cavia 10 (fig. 7) results in almost complete immunity from vaccine. Of all the

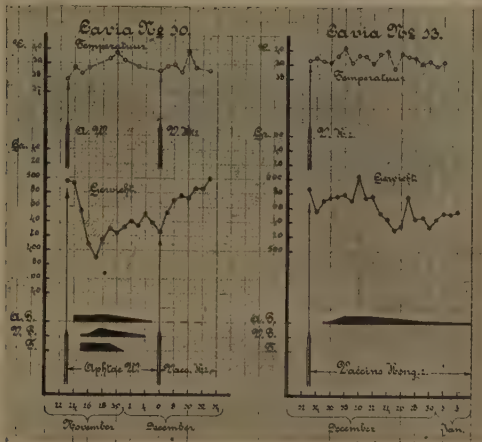


Fig. 7.

symptoms mentioned under II the only ones we notice here, is a rise of the temperature and very insignificant symptoms at the site of inoculation on the 4th day after the vaccine-inoculation. The graph of the control-cavia N^o. 13 shows that the inoculation-matter was sufficiently virulent (Fig. 7).

The same symptoms are distinguished in Cavia 4. This animal had been infected twice with aphthae, the first time with the slightly virulent strain S., the second time with the highly virulent strain W. After inoculation with "vaccine" the animal hardly displays any symptoms: we only detect a very slight reaction at the site of injection, which lasts two days (see fig. 8).

The control-cavia N^o. 12, however, reveals with the same infectious matter the complete aspect of the disease described in II.

¹⁾ See note p. 543.

This almost complete immunity could be established for three different vaccine-strains. The results of these experiments are illustrated in the subjoined table I.

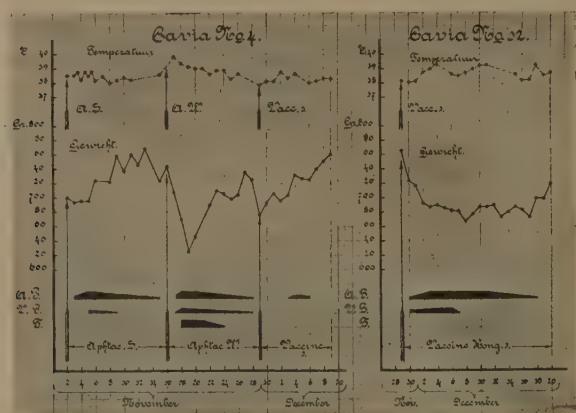


Fig. 8.

In one cavia the immunity appeared to be less complete. Here it should be remembered that this animal:

- 1°. could endure only faintly infectious aphthae,
- 2°. was already rather old, so that perhaps it had almost lost the capacity of appropriating this immunity.

This experiment, however, cannot on any account be brought forward to detract much from the significance of the other nine observations.

CONCLUSION.

The above-described results are significant for the following reasons.

The close relation between the aphthae- and the vaccine-virus is proved for the first time by scientific experiments.

The immunity from the latter, after inoculation with the former, points to a relationship between these two viruses.

It matters little that in these experiments vaccine-inoculation does not produce immunity against aphthae, as we could also establish that inoculation with different "aphthae-strains" did not yield immunity against other virulent foot- and -mouth disease-virus. Nevertheless, many facts point to a relationship between the two viruses.

This puts the observations of practitioners in a different light from that in which they were regarded up to the present.

On the basis of the relationship alluded to above, it may very well be possible that an intracutaneous vaccine-injection, properly carried out,

would yield a sufficient immunity against "contact"-infections with "aphthae-virus" to a degree that would make it serviceable for practice.

TABLE NO. 1.

Nº.	Weight	1st Inoculation	2d Inoculation	Vaccine	Résumé
8	860	Apht. W.		Hong. II	Almost complete immunity
9	500	Apht. W.		Hong. II	Almost complete immunity
19	420	Apht. W.		Hong. II	Very high degree of immunity
11	565	Apht. W. cavia 10		Hong. II	Very high degree of immunity
17	620	Apht. W. cavia 15		Amsterdam	Very high degree of immunity
18	900	Apht. W. cavia 17		Amsterdam	High degree of immunity
3	380	Apht. W.	Apht. W.	Hong. I	Almost complete immunity
4	700	Apht. S.	Apht. W.	Hong. I	Almost complete immunity
7	575	Apht. V.	Apht. W.	Hong. II	Almost complete immunity
6	680	Apht. S. cavia 4	Apht. W.	Hong. II	High degree of immunity

In connection with these experiments it is quite expedient that they should be continued in the direction indicated ¹⁾.

*From the Hygienic Institute of the Veterinary Faculty
of the Utrecht State-University.*

¹⁾ In two advisedly arranged experiments it could be established, that two caviae treated beforehand with vaccine and inoculated two, resp. four months afterwards with very virulent aphthae-virus, possessed a much greater resistance against this virus than the animals that had not been treated previously. If this finding should be confirmed in other cases, it will be recorded in a following communication.

Mathematics. — *A Representation on R_8 of the Projectivities of Point-Fields and Ray-Fields.* By Prof. H. J. VAN VEEN. (Communicated by Prof. W. VAN DER WOUDE).

(Communicated at the meeting of March 26, 1927).

§ 1. If in the relations:

$$\left. \begin{aligned} \varrho x'_1 &= a_1 x_1 + a_2 x_2 + a_3 x_3 \\ \varrho x'_2 &= a_4 x_1 + a_5 x_2 + a_6 x_3 \\ \varrho x'_3 &= a_7 x_1 + a_8 x_2 + a_9 x_3 \end{aligned} \right\} \dots \dots \dots (1)$$

the x_i and the x'_i ($i=1, 2, 3$) are considered as homogeneous point coordinates with respect to the same triangle of reference Δ in the plane τ , (1) associates a point x'_i to any point x_i of τ . We shall say of the set $[x'_i]$ of the points x'_i that through (1) it is in *projective* (*collinear*) correspondence with the point-field $[x_i]$. If the determinant of the 2nd members of (1) $|a| \neq 0$, also $[x_i]$ is projective with $[x'_i]$.

If the coordinates x_i of a point S make the 2nd members of (1) simultaneously zero, the associated point of S is indefinite; to such a *singular* point S we shall associate all points of τ .

The equations which we get by equating the 2nd members of (1) to zero, define 3 straight lines. They have either no common point or one common point S or they coincide in a line s . In the first case there is no singular point. In the second case S is singular; in this case we shall call the *projectivity* (*collineation*) *singular of the first kind*. In the third case any point of s is singular; let the line s be called a *singular line* and the projectivity *singular of the 2nd kind*.

For all singularities $|a| = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = 0$; for that of the 2nd kind at

the same time $\|a\| = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = 0$.

For a *singular collineation of the 1st kind* it is possible to choose the constants α , β and γ such that $\alpha(a_1 x_1 + a_2 x_2 + a_3 x_3) + \beta(a_4 x_1 + a_5 x_2 + a_6 x_3) + \gamma(a_7 x_1 + a_8 x_2 + a_9 x_3) \equiv 0$, hence for any point x'_i : $\alpha x'_1 + \beta x'_2 + \gamma x'_3 = 0$ or: to an arbitrary point x_i there corresponds a point x'_i of a fixed straight line; we shall call this the *singular line* s' of $[x'_i]$.

In a *singular collineation of the 2nd kind* the 2nd members of (1) are the same except for a constant factor; the ratios of the x'_i are, therefore, constant, or: to an arbitrary point x_i a fixed point is associated, the singular point S' of $[x'_i]$.

In what follows we shall indicate the singular collineations by (S, s') and (s, S') .

§ 2. I consider the 9 coefficients a_j ($j = 1 \dots 9$) of (1) as homogeneous coordinates of a point in an 8-dimensional space R_8 . To each collineation there corresponds a definite point of R_8 and inversely: each point of R_8 defines a projective correspondence of the points x'_i and x_i (also the direction of the correspondence).

To a system of ∞^m collineations there corresponds an m -dimensional variety V_m of R_8 . A similar variety is defined by nine equations of the form $a_j = f_j(p_1 \dots p_m)$ or by $8-m$ homogeneous equations of the form $F(a_j) = 0$; in the latter case we may consider V_m as the intersection of $8-m$ hypersurfaces. In a following paper we shall deal more elaborately with *systems of collineations*; now we shall only mention that through linear equations a linear space in R_8 is defined to which a linear system of collineations corresponds; to a straight line (a plane) of R_8 there corresponds a linear system of ∞^1 (∞^2) collineations, a *pencil* (*net*).

§ 3. The equation of the *locus of the image-points of the singular collineations* is:

$$|a| = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = 0$$

It is, therefore, a cubic hypersurface V_7^3 .

We remark that we get this equation by eliminating the parameters out of the equations

$$\left. \begin{aligned} \alpha a_1 + \beta a_2 + \gamma a_3 &= 0 \\ \alpha a_4 + \beta a_5 + \gamma a_6 &= 0 \\ \alpha a_7 + \beta a_8 + \gamma a_9 &= 0 \end{aligned} \right\}$$

or out of

$$\left. \begin{aligned} \lambda a_1 + \mu a_4 + \nu a_7 &= 0 \\ \lambda a_2 + \mu a_5 + \nu a_8 &= 0 \\ \lambda a_3 + \mu a_6 + \nu a_9 &= 0 \end{aligned} \right\}$$

so that V_7^3 can be built in two ways of the five-dimensional linear spaces of intersection of homologous elements of three projective nets of hyperplanes.

Any point P of V_7^3 is the image of a singular collineation of the 1st kind with singular point S and singular line s' . In order to investigate the system of all the collineations that have S as singular point, we

consider S as the angular point $0/0/1$ of the triangle of reference (change of the system of coordinates gives rise to a projective transformation in R_8). In our relations (1) we must now choose the coefficients such that for $x_1=0$, $x_2=0$ and $x_3=1$ the 2nd members all become zero; hence $a_3=a_6=a_9=0$. These are the equations of three hyperplanes which together define a five-dimensional linear space R_5 that passes through P and lies on V_7^3 .

In order to investigate the system of the collineations that have s' as singular line, we consider s' as side $x_3=0$ of the triangle of reference. Now in (1) we must find $x'_3=0$ for any system of values of the x_i ; hence $a_7=a_8=a_9=0$. Accordingly in R_8 we again find a five-dimensional space R'_5 through P lying on V_7^3 . R_5 and R'_5 have an R_3 as intersection, which is the image of all collineations that have S as singular point and s' as singular line: V_7^3 contains 2 systems of ∞^2 five-dimensional linear spaces $[R_5]$ and $[R'_5]$; the spaces R_5 are images of degenerate collineations with a fixed singular point S , the spaces R'_5 of degenerate collineations with a fixed singular line s' . Through any point of V_7^3 there passes one individual of each system (these spaces correspond to those mentioned in the 2nd paragraph of this section).

§ 4. The tangent hyperplane R_7 at a point P of V_7^3 contains the R_5 and the R'_5 that pass through this point, and is defined by them. At every point of the common space R_3 of R_5 and R'_5 V_7^3 has the same tangent hyperplane, hence: V_7^3 is touched by its tangent hyperplanes along an R_3 and for this reason we might call it a developable hypersurface of the 3rd kind.

Any R_5 (R'_5) is carrier of a net $[R_7]$ ($[R'_7]$) of ∞^2 tangent hyperplanes; all the tangent hyperplanes can be united in 2 ways to a system of ∞^2 nets.

§ 5. A singular collineation of the 2nd kind is defined by its singular line s and its singular point S' . There exist, therefore, ∞^4 such collineations; their image is a 4-dimensional variety, which can be fixed by the equations:

$$\|a\| = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = 0.$$

The order of this variety is the number of its points of intersection with an R_4 , which we can define by 4 linear equations:

$$\sum p_j a_j = 0, \quad \sum q_j a_j = 0, \quad \sum r_j a_j = 0 \quad \text{and} \quad \sum s_j a_j = 0.$$

If we put:

$$\frac{a_1}{a_4} = \frac{a_2}{a_5} = \frac{a_3}{a_6} = \frac{1}{\lambda}$$

and

$$\frac{a_1}{a_7} = \frac{a_2}{a_8} = \frac{a_3}{a_9} = \frac{1}{\mu}$$

we have:

$$(p_1 + p_4 \lambda + p_7 \mu) a_1 + (p_2 + p_5 \lambda + p_8 \mu) a_2 + (p_3 + p_6 \lambda + p_9 \mu) a_3 = 0$$

$$(q_1 + q_4 \lambda + q_7 \mu) a_1 + (q_2 + q_5 \lambda + q_8 \mu) a_2 + (q_3 + q_6 \lambda + q_9 \mu) a_3 = 0$$

$$(r_1 + r_4 \lambda + r_7 \mu) a_1 + (r_2 + r_5 \lambda + r_8 \mu) a_2 + (r_3 + r_6 \lambda + r_9 \mu) a_3 = 0$$

$$(s_1 + s_4 \lambda + s_7 \mu) a_1 + (s_2 + s_5 \lambda + s_8 \mu) a_2 + (s_3 + s_6 \lambda + s_9 \mu) a_3 = 0$$

If we consider λ and μ as non-homogeneous point coordinates, a_1 , a_2 and a_3 as parameters, these are the equations of 4 projective fields of rays. Now it happens 6 times that 4 corresponding lines pass through one point; hence our equations admit of 6 solutions: *the image of the singular collineations of the 2nd kind is a four-dimensional variety of the sixth order, V_4^6 .*

§ 6. A point P on V_4^6 is the image of a singular collineation of the 2nd kind (s, S') . We shall first consider all collineations that have s as singular line. With a view to this we choose s as side $x_3 = 0$ of the triangle of reference. As for any system of values of the x_i where $x_3 = 0$, all x'_i must be zero, in this case we find in (1), $a_1 = a_2 = a_4 = a_5 = a_7 = a_8 = 0$. Consequently these collineations have as image a plane α through P lying on V_4^6 . In a similar way we find that all singular collineations of the 2nd kind that have the point S' as singular point, are represented by a plane α' on V_4^6 ; V_4^6 contains 2 systems of ∞^2 planes $[\alpha]$ and $[\alpha']$; the α -planes are images of the singular collineations with a fixed singular line s , the α' -planes of the collineations with a fixed singular point S' . Through any point of V_4^6 there passes one plane of each of the two systems; two planes of the same kind have no point in common, two planes of different kinds have one point in common.

§ 7. Let $a_1 a_2 a_3$ be three planes of the 1st kind, α' an arbitrary plane of the 2nd kind, $S_1 S_2$ and S_3 the points of intersection of α' with $a_1 a_2$ and a_3 , and let S' be the common singular point of the collineations represented by α' . The fields of points $[S_1]$, $[S_2]$ and $[S_3]$ are in a $[1,1]$ correspondence with the field of points $[S']$ in τ and with each other. Let α_1 be the image of all collineations that have the line s as singular line. Choose s again as $x_3 = 0$; then the latter collineations are defined by the relations:

$$\left. \begin{aligned} \varrho x'_1 &= a_3 x_3 \\ \varrho x'_2 &= a_6 x_3 \\ \varrho x'_3 &= a_9 x_3 \end{aligned} \right\}$$

The singular point S' has for coordinates $a_3/a_6/a_9$. To any point S' in τ there corresponds an α' -plane, which cuts α_1 in a point S_1 . If S' moves on a line $p_1 x_1 + p_2 x_2 + p_3 x_3 = 0$, S_1 moves on the line of intersection of the plane α_1 with the hyperplane $p_1 a_3 + p_2 a_6 + p_3 a_9 = 0$; consequently the point-fields $[S_1]$, $[S_2]$ and $[S_3]$ are each in projective correspondence with the field $[S']$, hence also with each other, or: V_4^6 is the locus of the planes through the homologous points of three projective fields of points. As also the α -planes cut three arbitrary α' -planes in projective point fields, V_4^6 can be produced in two similar ways.

Now we may consider V_7^3 as built (§ 3) of the 5-dimensional linear spaces along which the homologous elements of 3 projective nets of hyperplanes cut each other; consequently V_7^3 and V_4^6 are dualistic figures: V_7^3 is of the 6th class and V_4^6 of the 3rd class, V_7^3 has ∞^4 tangent hyperplanes 6 of which pass through an R_3 , V_4^6 has ∞^7 tangent hyperplanes (in each point ∞^3) 3 of which pass through an R_6 .

§ 8. The polar hyperplane of the point $P(a_j^0)$ relative to V_7^3 has for equation $\sum a_j A_j^0 = 0$; here the A_j^0 are the coefficients of the elements a_j in the development of the determinant $|a|$ when the a_j have been replaced by the a_j^0 . If we choose P on V_4^6 the A_j^0 all become zero, the polar hyperplane is indefinite, hence P is a double point of V_7^3 ; V_4^6 is a double variety of V_7^3 .

The tangent hyperplane R_7 at a point P of V_7^3 passes through an R_5 and an R_5' (§ 4). R_5 (S fixed and s' arbitrary) contains ∞^1 α -planes: the images of the collineations that have as singular line s a ray of the plane pencil about S ; R_5' (s' fixed and S arbitrary) contains ∞^1 α' -planes: the images of the collineations that have a singular point S' on s' . In order to determine the locus of the points of intersection of these α - and α' -planes, we choose the line s' as side $x_3 = 0$ and the point S as angular point 0/0/1 of the triangle of reference. The relations of the collineation are in this case:

$$\begin{cases} \varrho x'_1 = a_1 x_1 + a_2 x_2 \\ \varrho x'_2 = a_4 x_1 + a_5 x_2 \\ \varrho x'_3 = 0 \end{cases}$$

where $\frac{a_1}{a_4} = \frac{a_2}{a_5}$, and $a_3 = a_6 = a_7 = a_8 = a_9 = 0$.

The locus in question is, therefore, a quadratic surface V_2^2 . Let P be a point of V_2^2 ; the α - and the α' -plane through P define the tangent space R_4 at P ; any hyperplane through R_4 touches V_4^6 at P , hence: the tangent hyperplanes of V_7^3 are double tangent hyperplanes of V_4^6 ; the points of contact of such a hyperplane form a quadratic surface.

§ 9. We shall now consider the system of the singular collineations

of the 1st kind where the singular point S lies on a given straight line. If we choose this line as side $x_3=0$ of the triangle of reference, the 2nd members of (1) for $x_3=0$ must agree except for a constant factor, hence:

$$\frac{a_1}{a_2} = \frac{a_4}{a_5} = \frac{a_7}{a_8}.$$

The image of these collineations, which consists of ∞^1 spaces R_5 , is, therefore, a six-dimensional cubic variety V_6^3 . V_7^3 contains ∞^2 such varieties; through any two points of the hypersurface there passes one. Each of the systems of collineations under consideration contains all the singular collineations of the 2nd kind.

The latter is also the case for each of the systems of singular collineations where the singular line s' is a ray of a given plane pencil; these systems have as images ∞^2 varieties V_6^3 on V_7^3 , hence: V_7^3 contains 2 systems of ∞^2 six-dimensional cubic varieties $[V_6^3]$ and $[V_6^3]$; the varieties of the 1st kind are images of all the singular collineations where S lies on a given straight line, those of the 2nd kind of the singular collineations where s' passes through a given point. All these varieties pass through V_4^6 .

§ 10. Through the relations

$$\left. \begin{aligned} \sigma \xi'_1 &= a_1 \xi_1 + a_2 \xi_2 + a_3 \xi_3 \\ \sigma \xi'_2 &= a_4 \xi_1 + a_5 \xi_2 + a_6 \xi_3 \\ \sigma \xi'_3 &= a_7 \xi_1 + a_8 \xi_2 + a_9 \xi_3 \end{aligned} \right\} \dots \dots \dots (2)$$

where ξ_i and the ξ'_i are considered as homogeneous ray coordinates relative to the triangle of reference Δ (§ 1), a line ξ'_i is associated to any line ξ_i of τ and if $|a| \neq 0$ also a line ξ_i to any line ξ'_i . The system $[\xi'_i]$ of the rays ξ'_i is again in a projective correspondence with the field of rays $[\xi_i]$ through (2).

Of these projectivities of rays we may make similar considerations as of projectivities of points.

We shall now consider the a_j as homogeneous coordinates of a hyperplane in R_8 . To the singular projectivities of rays of the 1st kind — i.e., accordingly, the projectivities through which to any ray ξ_i a definite ray ξ'_i (ray of the plane pencil about a point Σ') is associated except to one singular ray σ of which the associated line is indefinite — there corresponds a system of ∞^7 hyperplanes ${}^3W_7$ 3 of which pass through an R_6 and which has as equation:

$$|a| = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = 0.$$

To the singular projectivities of the 2nd kind — in which to any ray ξ_i one fixed ray ξ'_i is associated, which will be indicated by σ' , except

to the rays of the plane pencil about a point Σ for which the associated ray is indefinite — there corresponds a system of ∞^4 hyperplanes ${}^6W_4$ 6 of which pass through an R_3 .

§ 11. From the point-projectivity where the coefficients in (1) have the values a_j^0 , there generally follows a projectivity of rays where the coefficients in (2) are equal to A_j^0 and the A_j^0 have the signification indicated in § 8.

If P is the image of the point-projectivity, the polar hyperplane π of P relative to V_7^3 has for equation $\Sigma a_j A_j^0 = 0$, hence for coordinates A_j^0 , or: a point (hyperplane) and a hyperplane (point) are images of a point-(ray-) projectivity and of the corresponding ray- (point-) projectivity if they are pole (hyperplane) and polar hyperplane (pole) relative to V_7^3 (${}^3W_7$).

If P lies on V_7^3 and is, therefore, the image of a singular point-projectivity of the 1st kind with singular point S and singular line s' , π is the tangent hyperplane at P and the corresponding ray-projectivity is singular of the 2nd kind. For to an arbitrary line as carrier of its points there corresponds the fixed line s' and for the associated line of a line ξ_i through S any line of τ may be chosen, because any point of τ is associated to the point S of the line ξ_i (such in deviation from what is assumed by SEGRE, HIRST, STURM a.o.¹⁾). If the point-projectivity is defined by the coefficients a_j^0 , also in this case the coefficients for the ray-projectivity are A_j^0 . Accordingly the tangent hyperplanes of V_7^3 are identical with the hyperplanes of the system ${}^6W_4$; in the same way the variety enveloped by the hyperplanes of ${}^3W_7$ is identical with V_4^6 .

We still remark that a singular point- (ray-) projectivity of the 2nd kind does not give rise to a ray- (point-) projectivity.

§ 12. Summarizing we find:

If in the indicated way the point-projectivities are represented on the points and the ray-projectivities on the hyperplanes of an R_8 , R_8 contains a hypersurface V_7^3 of the 3rd degree the ∞^7 points of which are images of singular point-projectivities; this hypersurface is of the 6th class; its ∞^4 tangent hyperplanes are images of singular ray-projectivities of the 2nd kind. V_7^3 contains a double variety V_4^6 of the 6th order; its ∞^4 points are images of singular point-projectivities of the 2nd kind; this variety is of the 3rd class; its ∞^7 tangent hyperplanes are images of singular ray-projectivities of the 1st kind.

¹⁾ C. SEGRE. Sulla teoria e sulla classificazione delle omografie in uno spazio lineare ad un numero qualunque di dimensioni. Atti dei Lincei, 1884, (5), 19, 127.

T. A. HIRST. On the correlation of two planes. Proc. London Math. Soc. 5, 42.

R. STURM. Die Lehre v. d. Geometr. Verwandtschaften. 2, 219.

Mathematics. — *Systems of Plane Collineations.* By Prof. H. J. VAN VEEN. (Communicated by Prof. W. VAN DER WOUDE).

(Communicated at the meeting of April 30, 1927).

§ 1. Making use of "A Representation on R_8 of the Projectivities of Point-Fields and Ray-Fields" ¹⁾ in the following paper I shall derive properties of systems of collinear point-fields. (I¹⁾, § 2).

By a system of ∞^m collineations to the field $[x_i]$, the *identical field* or the *auxiliary field*, ∞^m point-fields are associated, which in what follows we shall indicate by $[A']_m$. As a rule any of these fields is in a projective correspondence with any of the others and that by means of their relation to the auxiliary field; in this way we arrive at a system of ∞^{2m} projectivities.

If we make the condition that a collineation must associate a point of the given line $a'(\sum \xi_i x_i = 0)$ to a given point $A(x_i^0)$, the a_j (I, § 2) must satisfy the equation:

$$\xi_1(a_1 x_1^0 + a_2 x_3^0 + a_3 x_3^0) + \xi_2(a_4 x_1^0 + a_5 x_2^0 + a_6 x_3^0) + \xi_3(a_7 x_1^0 + a_8 x_2^0 + a_9 x_3^0) = 0 \quad (1)$$

and inversely.

Accordingly the image of the collineations that satisfy the condition, is a hyperplane of R_8 .

Now among others there belong to this system of collineations:

1. all collineations that have A as singular point S (I, § 1 in connection with II²⁾, equation (1)); the image of these is an R_5 of V_7^3 (I, § 3);
2. all collineations that have a' as singular line s' ; these have as image an R_5' on V_7^3 (I, § 3).

R_5 and R_5' have the R_3 in common that represents the collineations with A as singular point S and a' as singular line s' ; they define a hyperplane (I, § 4): the collineations through which a point of the line a' is associated to A — including the singular collineations where A is the singular point S — have as image the hyperplane that touches V_7^3 along the R_3 which represents the singular collineations that have A as singular point S and a' as singular line s' .

If R_5 (R_5') is the image of all collineations that have A (a') as singular point (singular line), any hyperplane of the net (I, § 4, 2nd par.) about R_5 (R_5') is the image of the collineations for which the associated point of A either lies on a given line (the associated point of a given point lies on a') or is indefinite.

¹⁾ These Proceedings 30, p. 552.

²⁾ This paper.

§ 2. Through a point of R_8 there pass ∞^3 tangent hyperplanes of V_7^3 ; accordingly for *one collineation* it happens ∞^3 times that the associated point of a point x_i lies on a given line.

If the associated points of 8 given points A_k ($k=1, \dots, 8$) must lie on 8 given lines a'_k , we get in R_8 eight tangent hyperplanes of V_7^3 as images of these conditions. As a rule these have one point in common so that there is *one collineation* that satisfies the condition.

If the 8 hyperplanes have a line or a linear space of more dimensions in common, we might call the conditions *dependent*; in this case a linear system of collineations satisfies the conditions.

§ 3. Two points P_1 and P_2 of R_8 define a line R_1 , two collineations define a *linear system of ∞^1 collineations, a pencil*.

Through both points P_1 and P_2 , hence also through the line R_1 , there pass ∞^2 tangent hyperplanes of V_7^3 , one of any net of the two systems formed by these hyperplanes (I, § 4, 2nd par.). Consequently (II, § 1, last par.) for all collineations of the pencil the associated points of any point of τ lie on one straight line and any line of τ contains all the associated points of one point. (For the indefinitenesses we refer to II, § 9).

If we make the condition that *the associated points of 7 given points must lie on 7 given lines*, the image in R_8 of the collineations that satisfy this condition, is the common space of 7 tangent hyperplanes of V_7^3 , hence as a rule a straight line. Accordingly these collineations form a *pencil*.

Concerning the singular collineations we remark that a *pencil contains three singular collineations of the 1st kind*, as the image R_1 of a pencil has 3 points P_i in common with V_7^3 ; two or three of these collineations can coincide. If R_1 cuts V_4^6 the pencil contains a *singular collineation of the 2nd kind and one of the 1st kind*. If R_1 is a bisecant of V_4^6 it lies on V_7^3 , hence: *if a pencil contains two singular collineations of the 2nd kind, it consists entirely of singular collineations*.

§ 4. Three points of R_8 that do not lie on one line, define a plane R_2 , three collineations that do not belong to one pencil, define a *linear system of ∞^2 collineations, a net*. Through R_2 there pass ∞^1 tangent hyperplanes of V_7^3 , hence there are ∞^1 points x_i of which the associated points lie on one line. (For the indefinitenesses, the 5th par. of this §).

R_2 cuts V_7^3 along a curve of the 3rd order k : a *net contains ∞^1 singular collineations of the 1st kind, but as a rule no singular collineations of the 2nd kind*.

The plane R_2 is cut by each of the varieties $V_6^3 (V_6^3) - I$, § 9 — in 3 points, hence: *in a net of collineations the locus of the singular points S is a curve of the 3rd order k^3 , the singular lines s' envelop a curve of the 3rd class ^3k* .

Let A be a point of τ that is not singular for any of the collineations of our net, R_5 the image of all collineations that have A as singular point S , then R_5 has no point in common with our R_2 . Accordingly none of the tangent hyperplanes through R_5 of V_7^3 contains R_2 . Through any line of R_2 there passes one of the hyperplanes of each net $[R_7]$: for each of the ∞^2 pencils in our net A' describes one of the rays of the field in τ . To any point P of R_2 as image of a collineation one point A' of τ corresponding to A is associated and inversely; if P describes a line A does the same; the fields of points $[P]$ and $[A']$ are projective.

We shall now consider a point S of τ that is singular for one of the collineations of our net. In this case the R_5 mentioned in the preceding section has one point, lying on k , in common with R_2 and one of the hyperplanes through R_5 contains R_2 . The associated points of S lie, therefore, as a rule on one line. If s' is a singular line for any of the collineations under consideration, the R_5 (s' singular) has also a point in common with R_2 so that through this R_5 there also passes an R_7 that contains R_2 : all the associated points of a point S of the curve k^3 — except for the collineation for which S is a singular point, where there is an indefiniteness — lie on a tangent s' to the curve ^3k and inversely: any tangent s' to ^3k contains the associated points of a point S of k^3 .

The collineations for which the associated points of 6 given points lie on 6 given lines (or are indefinite), generally form a net, for the image of these collineations is the plane of intersection of 6 tangent hyperplanes of V_7^3 .

§ 5. Four points of R_8 that do not lie in one plane, define an R_3 , four collineations that do not belong to one net, define a linear system of ∞^3 collineations, a linear complex.

Through an R_3 (I, § 7, last paragraph) there pass six tangent hyperplanes of V_7^3 , hence: a linear complex of collineations contains six points for each of which the associated points are collinear. (See below the cases of indefiniteness).

Now all collineations for which all the associated points of 5 given points lie on 5 given lines, form a linear complex, because the image in R_8 is the space of intersection R_3 of 5 tangent hyperplanes of V_7^3 , hence: in all the collineations for which the points associated to five given points lie on five given lines, there is a sixth point for which the associated points are also collinear¹⁾. However we must remark that our R_3 has a point in common with any R_5 so that any point x_i of τ is singular for one of the collineations in question and its associated point indefinite. Also any line of τ is the singular line s' for one of these collineations. Moreover our R_3 has a line in common with the

¹⁾ A. CLEBSCH, Ueber ein neues Grundgebilde der analytischen Geometrie der Ebene. Math. Annalen 6, p. 203, 1873.

J. ROSANES, Ueber linear abhängige Punktsysteme. Journ. f. Mathem. Bd. 88, p. 241, 1880.

A. VOSS, Zur Theorie der linearen Connexe. Math. Ann. 15, 355, 1879.

R. STURM, Ueber Collineation und Correlation. Math. Ann. 22, 569, 1883.

R_5 (R'_5) with which it lies in the same tangent hyperplane of V_7^3 , hence: each of the six points of which the associated points generally lie on six fixed lines, is singular point S for a pencil of collineations; the six fixed lines are then the singular lines s' .

As a rule a linear complex does not contain any singular collineations of the 2nd kind. Such can be the case for special linear complexes.

§ 6. Five points that do not lie in one R_3 , define an R_4 , five collineations that do not belong to a linear complex, define a linear system of ∞^4 collineations.

Through an arbitrary R_4 there do not pass any tangent hyperplanes of V_7^3 , so that as a rule it does not happen in a linear complex that all the associated points of one point are collinear; in special cases, however, this is possible; the collineations for which the associated points of 4 given points lie on 4 given lines, (or are indefinite), form a — special — linear system of ∞^4 collineations.

In a linear system of ∞^4 collineations any point (any line) of τ is singular for a pencil of collineations.

An R_4 has 6 points of intersection with V_4^6 , hence: a linear system of ∞^4 collineations contains six singular collineations of the 2nd kind ¹⁾.

In the same way we can derive properties of linear systems of ∞^5 , ∞^6 and ∞^7 collineations.

§ 7. We shall now consider the ∞^1 collineations for which the associated points A'_l ($l = 1 \dots 5$) of 5 given points A_l lie on 5 given lines a'_l and the associated points B' of the point B on a curve k of the n^{th} order.

The condition that the points A'_l must lie on the lines a'_l , defines a net of collineations which has as image in R_8 a plane R_2 . In § 4 we have shown that for these collineations the associated points of any point of a certain cubic k^3 lie on a tangent to a curve of the third class ^3k .

At the same time it follows from the projectivity mentioned in the 4th paragraph of § 4, that the image of the collineations which satisfy all the conditions, is a curve projective with k ; consequently that all the associated points of any point of τ that is not singular for any of the collineations of the net, and which, therefore, does not lie on k^3 , lie on a curve projective with k . We also remark that our system contains $3n$ singular collineations, hence: if the associated points of 5 given points lie on 5 given lines and those of a sixth point on a curve k of the n^{th} order, τ contains a curve of the 3rd order k^3 with the following properties: k^3 contains $3n$ points for each of which the associated point is indefinite for one of the collineations in question. With the exception of these points all the associated points of any point of k^3 lie on a tangent to a curve of the third class ^3k . All the associated points of any other point of τ lie on a curve that is projective with k .

¹⁾ STURM, p. 581.

It follows from § 11 that as a rule τ does not contain any points to which in the collineations under consideration a fixed point is associated.

§ 8. The collineations for which the associated point of a given point A must lie on a given line a'_1 , have as image in R_8 an R_7 through R_5 representing the collineations for which A is singular point S ; the same is the case with the collineations for which the associated point of A lies on the line a'_2 . Accordingly the collineations for which A corresponds to the point of intersection A' of a'_1 and a'_2 , have an R_6 through the said R_5 as image; among them there are ∞^5 collineations for which the associated point of A is indefinite. If now we consider all fields of points $[A']_6$ that correspond to the auxiliary field (II, § 1) by means of our collineations, these projective fields have A' as *invariant point*.

Each of the ∞^2 spaces R_6 through R_5 is the image of the collineations that make A correspond to a given point of τ . We get a system of ∞^4 spaces R_6 , which may be combined to ∞^2 nets $[R_6]$ each of which has one of the ∞^2 spaces R_5 on V^3_7 (I, § 4) as carrier.

In a similar way we can derive the existence of a system of ∞^4 spaces R'_6 — these may be combined to ∞^2 nets $[R'_6]$ each of which has one of the spaces R'_5 (I, § 4) as carrier — that are the images of the collineations which associate a fixed line to a given line. In this case the point-fields corresponding to the auxiliary field have an *invariant line*.

Two of the said spaces R_6 (R'_6) have an R_4 (R'_4) in common, which is the image of all collineations that associate ∞^4 projective point-fields with *two invariant points (lines)* to the auxiliary field. In the same way three such spaces R_6 (R'_6) have a plane R_2 (R'_2) in common, which represents the collineations that associate ∞^2 projective point-fields with *three invariant points (lines)* to the auxiliary field. Four of the spaces R_6 (R'_6) generally have one point in common; there is only one collineation that associates 4 points (lines) of $[x]$ to 4 given points (lines).

If we also consider the intersections of spaces R_6 with spaces R'_6 we get the images of all *fundamental groups with invariant points and lines*¹⁾.

§ 9. Through a point P of R_8 there pass ∞^2 of the spaces R_6 (R'_6), one of each net; in fact for *one collineation* the element associated to an arbitrary point (arbitrary line) is fixed.

We shall now consider a *pencil of collineations*; this generally contains

¹⁾ LIE—SCHEFFERS, *Continuierliche Gruppen* p. 288.

H. B. NEWSON, A new theory of Collineations and their LIE-groups. *American Journal of Mathematics*, 24, p. 169, 1902.

H. B. NEWSON, Report on the theory of collineations. *Proceedings of the American Association*. Pittsburgh, 1902, p. 579.

3 singular collineations (II, § 3) with singular points A_i and singular lines a'_i ; their images in R_8 are the points of intersection P_i of the image R_1 of the pencil with V_7^3 .

Each of the points P_i lies in one R_5 (R'_5); this is the carrier of ∞^2 spaces R_6 (R'_6) one of which passes through R_1 : a pencil of collineations contains three points A'_i that have a fixed associated point A'_i — except in the collineations for which they are singular elements, where the associated point is indefinite —; in the same way there are three lines a_i to each of which a fixed line a'_i is associated; these lines a'_i are the singular lines of the 3 singular collineations of the pencil.

A pencil of projective point-fields contains three invariant points A_i and three invariant lines a'_i .

The R_5 through P_1 lies in one tangent hyperplane of V_7^3 with the R'_5 through P_2 (and likewise with the R'_5 through P_3); this hyperplane contains R_1 , hence: the points A'_i are the angular points of the triangle of which the sides lie on the lines a'_i .

§ 10. If the image line R_1 of a pencil is a tangent to V_7^3 , $P_{1,2}$ its point of contact and P_3 its point of intersection, the pencil contains a singular collineation with singular point $A_{1,2}$ and singular line $a'_{1,2}$ and a singular collineation with singular point A_3 and singular line a'_3 . To $A_{1,2}$ there corresponds a fixed point $A'_{1,2}$ which again lies on a'_3 but at the same time on $a'_{1,2}$, because R_1 lies in the tangent plane at $P_{1,2}$ to V_7^3 . The fixed point A'_3 associated to A_3 lies likewise on $a'_{1,2}$.

If the image line R_1 is an inflexional tangent to V_7^3 , the pencil contains only one singular collineation (A, a'); the point A' associated to A is fixed and lies on a' .

Invariant points and invariant lines of two projective point fields are also invariant elements of the pencil of projective fields defined by them. Making use of the notation of SEGRE¹⁾ we may, therefore, say: a collineation belongs to the type $[1, 1, 1]$, $[2, 1]$ or $[3]$ according as the line joining its image point to the image point of the identical collineation cuts V_7^3 in 3 different points or is a common tangent or an inflexional tangent of the hypersurface. (Cf. further § 12).

§ 11. As a rule through a plane there does not pass any of the spaces R_6 (R'_6), hence: a net of collinear point-fields does not generally contain any invariant point. If, however, the plane lies in one of the spaces R_6 (R'_6), it is the image of the collineations that associate a net of projective point-fields with one invariant point (line) to the auxiliary field. A plane in the common space R_4 of 2 spaces R_6 (R'_6) associates a net of projective fields with two invariant points (lines) to the auxiliary field. A net with 3 invariant points has already been mentioned in § 8 and

¹⁾ C. SEGRE, Sulla teoria e sulla classificazione delle omografie in uno spazio lineare ad un numero qualunque di dimensioni. Atti dei Lincei, 1884, (5), 19, 127.

from the discussions there we can derive all possibilities — concerning the invariant elements — for linear systems of ∞^3 , ∞^4 , ∞^5 and ∞^6 projective fields.

§ 12. If P_1 is a point of intersection of an arbitrary line R_1 with V_7^3 , only one of the spaces R_5 (R_5') passes through P_1 and only one of the spaces R_6 (R_6') of the net (II, § 8) that has this R_5 (R_5') as carrier.

If, however, R_1 cuts V_7^3 in a point P ($s=a$, $S'=A'$) of V_4^6 , this point lies in ∞^1 spaces R_5 (S on a) and in ∞^1 spaces R_5' (s' through A'); each of these R_5 (R_5') is the carrier of a space R_6 (R_6') through R_1 . Consequently we have in our collineations (cf. II, § 9): to any point of the line a there corresponds a fixed point of a line a' and to any line of a plane pencil about a point A there corresponds a fixed ray of the plane pencil about A' ; A is the singular point S and a' is the singular line s' for the collineation that has the 2nd point of intersection of R_1 and V_7^3 as image: if a pencil of collineations contains a singular collineation of the 2nd kind (s , S'), the point fields $[A']_1$ are generally central-collinear (perspective); in this case the pencil has also one degeneration of the 1st kind (S , s'); the singular elements S' and s' are the centre and the axis of these perspectivities.

If R_1 is a tangent to V_7^3 at a point of V_4^6 , there correspond to this image-line perspectivities in which the centre lies on the axis.

In connection with § 10, last paragraph, we remark: a collineation belongs to the type $[(1,1), 1]$ or to the type $[(2,1)]$ according as the line that joins its image point to the image point of the identical collineation, cuts V_4^6 or touches V_7^3 at a point of V_4^6 .

§ 13. If by a central-collineation we understand a collineation through which to the auxiliary field a field is associated that is central-collinear with it, and if we call the image point of the identical collineation H , we find: the image of all central-collineations is formed by the five-dimensional cone of the sixth degree K_5^6 that has H as vertex and V_4^6 as director variety.

An arbitrary R_3 cuts K_5^6 in six points; if R_3 passes through H it has only this point (which is sixfold for K_5^6) in common with the cone; as any field of the system of projective fields $[A']_3$ corresponding to R_3 may be chosen as auxiliary field, we have: a linear system of ∞^3 projective point-fields $[A']_3$ generally contains 6 fields that are perspective with the auxiliary field ¹⁾; but no two of these fields are perspective with each other.

An R_4 cuts V_4^6 in 6 points; a line of R_4 through one of these points defines a pencil of perspective point-fields: a linear system of ∞^4 projective point-fields contains 6 systems of ∞^3 pencils of perspective fields.

¹⁾ STURM p. 574.

The curves of Γ_{12} are represented in one, *singular*, point S_{12} , the pencils (μ^n) in the planes Db_k ($k=3$ to $2n$) in the *singular* points S_k . There are $\psi(n-1)$ more *singular* points S , each of which represents a pencil of which any individual consists of a fixed μ^{n-1} and a variable line.

3. The monoids resting on a line l form a surface Δ and are represented on a curve λ . Through a point B_1 of b_1 there pass $f(n)-n$, hence n^2 , curves μ^n ; λ is, therefore, the product of two plane pencils in a correspondence (n^2, n^2) , accordingly a λ^{2n^2} , with n^2 -fold points F_1 and F_2 .

It has an $n(n-1)$ -fold point in S_{12} and n -fold points in each of the $2(n-1)$ points S_k ; it also contains the $\psi(n-1)$ points S .

Any non singular point of intersection of two curves λ corresponding to the lines l and l' , is the image of a μ^n that rests on l and l' . As the number of these monoids is represented by $\psi(n)$ the relation

$$2n^4 + n^2(n-1)^2 + 2(n-1)n^2 + \psi(n-1) + \psi(n) = 4n^4$$

holds good, whence

$$\psi(n) + \psi(n-1) = n^2(n^2 + 1).$$

As $n^2(n^2 + 1) = \frac{1}{2}n^2(n+1)^2 + \frac{1}{2}(n-1)^2n^2$, it appears that

$$\psi(n) = \frac{1}{2}n^2(n+1)^2$$

Consequently on $2n+2$ lines there rest $\frac{1}{2}n^2(n+1)^2$ monoids that have their $(n-1)$ -fold point in D .

4. The monoids that rest on $2n$ lines b and touch a plane φ , form a surface Φ .

The monoid M of the order n^2 which contains the curves μ^n (D^{n-1}) through B_1 , has an $n(n-1)$ -fold line in DB_1 . Accordingly through the point of intersection of DB_1 and φ there pass $n^2(n^2-1)-n(n-1)(n^2-n+1)$ or $n(n-1)(2n-1)$ tangents to the intersection of M and φ . Through B_1 there pass, therefore, $n(n-1)(2n-1)$ curves μ^n which touch the plane φ ; consequently the image curve of the system of the curves touching φ has the order $2n(n-1)(2n-1)$.

The monoid Γ_{12} of the order $n(n-1)$ has an $(n-1)^2$ -fold line in a_{12} ; hence $n(n-1)(n^2-n-1)-(n-1)^2(n^2-2n+2)$ curves μ^{n-1} touching φ lie on Γ_{12} . Through the point of intersection of a_{12} and φ there pass $(n-1)^2$ monoids consisting of a_{12} and a μ^{n-1} , each of which has that point as a double improper point of contact with φ .

In all Γ_{12} contains, therefore, $(n-1)(n-2)(2n-1)+2(n-1)^2$ or $n(n-1)(2n-3)$ monoids touching φ ; hence the image curve of the system has an $n(n-1)(2n-3)$ -fold point in S_{12} .

The plane Db_k contains $2(n-1)$ monoids μ^n that touch φ ; as they cut b_k n times, S_k is a $2n(n-1)$ -fold point.

Each of the $\psi(n-1) = \frac{1}{2}(n-1)^2 n^2$ planes which contain a fixed μ^{n-1} and a pencil about D , gives rise to $n-1$ composite μ^n of our system that must be counted double; accordingly the corresponding singular point S is $2(n-1)$ -fold on the image curve. This may, therefore, be indicated by the symbol

$$\varphi^{2n(n-1)(2n-1)} (F_{1,2}^{n(n-1)(2n-1)}, S_{12}^{n(n-1)(2n-3)}, 2(n-1) S_k^{2n(n-1)}, \frac{1}{2}(n-1)^2 n^2 S^{2(n-1)})$$

It has as many non-singular points in common with the image curve

$$\lambda^{2n^3} (F_{1,2}^{n^3}, S_{12}^{n(n-1)}, 2(n-1) S_k, \frac{1}{2}(n-1)^2 n^2 S)$$

of the monoids that rest on l as the number of the μ^n , that touch φ and cut l .

Now

$$4n^3(n-1)(2n-1) - \{2n^3(n-1)(2n-1) + n^2(n-1)^2(2n-3) + 4(n-1)^2 n^2 + (n-1)^3 n^2\}$$

is equal to $n^3(n^2-1)$. Consequently there are $n^3(n^2-1)$ monoids that touch φ and cut l ; the monoids which touch φ and cut $2n+1$ lines, form a surface of the degree $n^3(n^2-1)$.

The combination of two image curves φ gives: there are $n^2(n-1)^2(2n^2-1)$ monoids with an $(n-1)$ -fold point in D which touch two planes and cut $2n$ straight lines.

Biochemistry. — *The so-called co-enzyme of alcoholic fermentation.* By
A. J. KLUYVER and A. P. STRUYK.

(Communicated at the meeting of April 30, 1927).

I. In 1905 it was shown by HARDEN and YOUNG¹⁾ that it is possible to divide zymase containing liquids by ultrafiltration into two parts each of which misses the power to ferment glucose, whereas by reuniting both this power is again restored. In the same year it was proved by BUCHNER and ANTONI²⁾ that a similar separation could be effected also by dialysis of yeast-juice. It soon appeared from further experiments that contrary to what counts for the remaining mass in the ultrafilter or in the dialysator, the filtrate and the dialysate could be heated to 100° C. without causing the least change in the observed effect. The possibility that the inactivity of the residue should have to be reduced exclusively to the removal of the soluble alkaliphosphates proved indispensable to fermentation, was excluded by further experiments of HARDEN. Hence this investigator concluded that for the fermentation of the hexoses not only the presence of the zymase proper — and of alkaliphosphates — was required, but also the presence of an ultrafiltrating, dialysable, thermostable substance, which he gave the name of “co-enzyme”³⁾.

The interest for this co-enzyme has risen considerably in the last years, since namely O. MEYERHOF proved in 1917 that the occurrence of this principle was by no means confined to the yeast-cell; on the contrary the co-enzyme was also found in various animal tissues. Moreover MEYERHOF made also acceptable that the co-enzyme was not only acting in fermentation but also in respiration processes.

Notwithstanding the fact that the co-enzyme has been the subject of many investigations in recent years, one has not yet succeeded in elucidating the nature of this so mysterious substance. So there are to be found assembled in recent summaries of our knowledge in this domain much varying and to a certain extent hardly combinable opinions about the nature of the co-enzyme⁴⁾.

Characteristic however for the mode of view of all investigators is that they have not the least objections to accept the necessity of the co-operation of an unknown factor for the realisation of the alcoholic fermentation of the

¹⁾ Compare: A. HARDEN, *Alcoholic Fermentation*, 3rd Ed., 1923, p. 61.

²⁾ E. BUCHNER und W. ANTONI, *Zeitschr. f. physiol. Chemie*, Bd. 46, p. 136, (1905).

³⁾ In literature the names co-enzyme, coferment, cozymase and codiastase are mixed up.

⁴⁾ For the sake of brevity this assertion will not be further documented here; compare the wellknown summaries of FUCHS, NEUBERG, HARDEN, VON EULER and SCHOEN.

hexoses. This cannot wonder when one realizes that biochemists are still generally used to reduce the chemical conversions, caused by the living cell, to a chain of reactions each of which passes under the influence of its own specific catalysator. Under these circumstances there is little objection to the acceptance of one single mysterious agent — the co-enzyme — more.

However this becomes different when one puts himself on the standpoint taken in by DONKER and one of us (KLUYVER)¹⁾ according to which the biochemical conversions — apart from some hydrolyses and esterifications, which are for the greater part of a preparatory nature — are to be reduced to a chain of reactions which may all be considered as a catalytic transference of hydrogen and which pass in each cell under the influence of one and the same agent.

Under these circumstances biochemistry is reduced for the greater part to a series of heterogeneous catalytic processes the model of which is to be found in organic chemistry in the coupled dehydrogenations and hydrogenations under the influence of finely divided nickel.

The simplification of insight brought along with this view renders the acceptance of the necessity of the collaboration of a co-enzyme for the fermentation process much more difficult and it seemed therefore desirable to us to consider in how far the necessity of an agent, besides the zymase proper, was to be derived from our special scheme for alcoholic fermentation based on the general theory referred to above.

II. For the scheme mentioned, we think we may refer to the last cited paper of KLUYVER and DONKER, and also to a former paper of ours²⁾. In view of the fact that an esterification namely the formation of a phosphoric ester occurs as the preliminary reaction, it seemed in the first place advisable to consider whether the co-enzyme was the agent, the phosphatase (resp. phosphatase) catalysing this reaction, resp. the hydrolysis of the triose phosphoric ester. Meanwhile this idea has been rejected at once on account of the fact that it is quite unacceptable to attribute the properties of easy dialysability and especially large thermostability to an enzyme such as the phosphatase³⁾.

However, a further consideration of the fermentation scheme leads to the conclusion that still another factor than only the zymase in its restricted sense and the phosphatase is required for the setting in of fermentation. As known the dehydrogenation of the methylglyoxal hydrate occurs in a normal way with acetaldehyde as acceptor whereby meanwhile the acetaldehyde is formed by the reaction ensuing therefrom (the decarboxylation of the pyruvic acid). So this situation implies the presence of a substance necessary for the commencement of fermentation, which

¹⁾ A. J. KLUYVER and H. J. L. DONKER, *These Proceedings* 28, 297, (1925); 28, 605, (1925) and *Chemie der Zelle und Gewebe*, Bd. 13, p. 134, (1926).

²⁾ *These Proceedings* 29, p. 322, (1926).

³⁾ As stated before it was already early shown by HARDEN that the effect of the co-enzyme does not rest exclusively upon the removal of the alkaliphosphates.

substance takes over the function of the at first missing acetaldehyde. This consideration made it possible that the co-enzyme should only act as a hydrogen acceptor and looking at this hypothesis we decided to approach experimentally the co-enzyme problem.

III. Before giving a short review of the result of our own experiments we must point out that some experiments were already made by NEUBERG ¹⁾ as well as by HARDEN ²⁾ the results of which could be interpreted at least partly in the described way. We will not discuss these results any further here, but will only remark that on the one side HARDEN ends his summary given in 1923 ³⁾ with the words: "It seems highly probable that the co-enzyme fulfils a similar function and it will be found like aldehyde, to be a substance capable of reduction in presence of the enzymes of yeast" whilst on the other side MEYERHOF, on the ground of his own experiments firmly denies the possibility of the substitution of the co-enzyme by aldehydes.

When making our own experiments we decided first of all to use the technic originally followed by HARDEN, with the difference, however, that maceration extract was used instead of BUCHNER's yeast-juice. Small quantities thereof were divided into two parts in an ultrafiltration apparatus according to BECHHOLD, after which the material remaining on the filter was washed with a little water. Further it was investigated how far the fermenting power of the inactive residue, which caused strong fermentation after addition of the ultrafiltrate, was also restored by adding small quantities of acetaldehyde accompanied by suitable quantities of alkali phosphates.

The results of a series of investigations in this direction have been varying; sometimes it has been possible indeed to lead an otherwise wholly negative remaining mixture to strong fermentation by the addition of acetaldehyde and sometimes this addition remained without any effect, although, even then activation by ultrafiltrate occurred. The nature of the yeast used for the preparation of the maceration extract as well as the duration of the filtration and the intensity of washing of the residue seemed to influence the result.

In the meantime it could be said with certainty that the function of the co-enzyme was not only that of an introductory hydrogen acceptor as otherwise substitution by acetaldehyde should have been possible in all cases.

IV. With this series of experiments we had purposely not yet taken into account a possible complication to which MEYERHOF ⁴⁾ drew the attention by his important investigations about the co-enzyme. Starting

¹⁾ C. NEUBERG und E. SCHWENK, *Biochem. Zeitschr.*, Bd. 71, p. 135, (1915).

²⁾ A. HARDEN, *Biochemical Journal*, Vol. 11, p. 64, (1917).

³⁾ A. HARDEN, *Alcoholic Fermentation*, 3rd Ed. London, 1923, p. 72.

⁴⁾ O. MEYERHOF, *Zeitschr. f. physiol. Chemie*, Bd. 101, p. 165, (1918), *Ibid.* 102, p. 1, (1918), *Ibid.* 102, p. 185, (1918).

from the phenomenon of induction shown by certain maceration extracts, discovered by LEBEDEFF ¹⁾, MEYERHOF points to the part which this phenomenon can play with experiments about the co-enzyme made with normal maceration extract.

We can summarise MEYERHOF's theories on this point by saying that he shows the necessity of making difference between the apparent inactivity of a zymase preparation — which is a consequence of the induction — and a real inactivity caused by the absence of the co-enzyme.

It is possible to distinguish between these two phenomena because of MEYERHOF's important discovery that the induction contrary to inactivity caused by lack of co-enzyme can be eliminated by addition of a small amount of hexosephosphate. So the result is that no zymase preparation, which does not remain inactive after addition of hexosephosphate, may be considered free from co-enzyme and MEYERHOF points out that to obtain this effect continued washing on an ultrafilter is indispensable.

Entirely in accordance with this it appeared to us from a new series of experiments — whereby we used like MEYERHOF the more simple ultrafiltrating apparatus of ZSIGMONDY — that the zymase preparations inactivated by too little washing, were only apparently inactive, in so far, that the addition of hexosephosphate ²⁾ was sufficient to bring about a strong fermenting power. Further we found also that continued washing to a 200 fold dilution of the materials accompanying the zymase — led indeed to a zymase preparation which could be activated by the usual co-enzyme solutions, such as boiled maceration extract, but not by hexosephosphate.

Thus it had now become necessary to repeat the experiments concerning the possibility of activation by acetaldehyde of a zymase preparation really freed from co-enzyme. These experiments, however, gave without exception a negative result, so that the conclusion had to be drawn that except the hexosephosphate, still another substance — the co-enzyme proper — is required to make the zymase capable of producing sugar fermentation.

V. Herewith the way to a closer understanding of the function of the co-enzyme seemed to be blocked up for the present. After ample deliberation it became however alluring to connect the problem with phenomena in a somewhat further off area which — although full attention was given to it by the first zymase investigators — had been left later on with a single exception, without any consideration. We allude to the fact that it was already proved by HAEHN in 1898 that zymase preparations always possess a strong proteolytic power and in connection herewith are also subject to a strong autolysis by which the zymase itself is also destroyed. Hereto were added the later observations of BUCHNER and his

¹⁾ A. LEBEDEFF, *Annales de l'Institut Pasteur*, T. 26, p. 8, (1912).

²⁾ When in this paper is spoken of hexosephosphate always the with natriumoxalate conversed "Candiolin BAYER" or the corresponding "hexosediphosphorsäures Natrium", from the same firm is meant. We want to express here also our best thanks to the named firm, which was so kind to put the mentioned preparations at our disposal.

collaborators, including a.o. HAEHN, who proved that boiled maceration extract possesses an undeniable antiproteolytic action¹⁾. The possibility lying at hand that the co-enzyme function of boiled zymase preparations should have to be reduced to the supposed presence of a so-called "antiprotease", in other words to the zymase protecting action of these juices was meanwhile rejected by the named investigators, a.o. after observation that by continued boiling the co-enzyme function can be destroyed, whilst the "antiprotease" function remains then for the greater part preserved. The same conclusion was drawn by HAEHN and SCHIFFERDECKER²⁾ who published in 1923 a detailed investigation on this point.

Now we specially noted the fact that whilst on the one side MEYERHOF in his former cited investigations does not pay any attention to the studies of BUCHNER and collaborators about the proteolytic function of the zymase preparations, on the other side HAEHN and SCHIFFERDECKER seem to have remained ignorant of the 6 years earlier by MEYERHOF published results and thus had not taken into consideration at all the according to MEYERHOF secondary influence of a possible want of hexosephosphate. This situation seemed to us to imply the possibility that the principle considered by BUCHNER and SCHIFFERDECKER as "co-enzyme" might not be anything else than hexosephosphate which is destroyed in small concentrations by continued boiling whereas the by MEYERHOF as "co-enzyme" (Coferment) indicated principle should have to be identified with the antiproteolytic factor present in the boiled juices.

Numerous observations in the series of experiments of the different investigators proved to be in perfect accordance with this hypothesis, but for shortness' sake we will not go further into this here.

An experimental test of the hypothesis given above led to a satisfactory result. It appeared to be possible indeed according to HAEHN and SCHIFFERDECKER to free a solution of co-enzyme by seven hours boiling entirely of its original activating power and to regenerate this by simple addition of some hexosephosphate. On the other hand a zymase preparation remaining inactive with hexosephosphate, could be activated by simple addition of the meant "Schutzsaft", so that there stood nothing in the way of identifying MEYERHOF's co-enzyme with BUCHNER's "antiprotease".

In the meantime as long as the nature of the antiproteolytically acting substances was not further ascertained, the above mentioned experiments did not exclude all doubts regarding the correctness of this last contention. It might be possible that besides these substances still another substance, the co-enzyme proper, had also endured without difficulty the long process of boiling, although this seemed improbable since it follows also from

¹⁾ E. BUCHNER und H. HAEHN, *Biochem. Zeitschr.*, Bd. 19, p. 198, (1909); *Ibid.* Bd. 26, p. 171, (1910).

²⁾ H. HAEHN und H. SCHIFFERDECKER, *Biochem. Zeitschr.*, Bd. 138, p. 209, (1923).

THOLIN's ¹⁾ experiments that the co-enzyme is practically entirely destroyed by heating to 100° C. during seven hours.

Meanwhile we did our best to consolidate our view concerning the nature of the co-enzyme of MEYERHOF. Therefore it was necessary to form a more precise idea regarding the nature of the antiproteolytic action. It seemed probable to ascribe this action to proteins which might act as substrates of the endotryptase and in this way function to a certain extent as "lightning conductor" for the zymase. However great difficulties were in the way of an experimental test of this hypothesis, as protein extracts from much varying vegetable and animal organs were just tested by MEYERHOF on their activating function with positive result, where from this investigator concluded to the very general occurrence of the "co-enzyme".

Thus it was of importance for us to find a protein containing substance which was beyond any suspicion of containing MEYERHOF's co-enzyme. Such a material was only to be found, in the form co-enzyme, according to MEYERHOF's prescriptions, freed zymase. It seemed probable that a zymase preparation free of co-enzyme and subjected to a moderated autolysis after boiling up, in other words after destroying the zymase and the endotryptase should contain smaller complexes of proteins which in a second portion of zymase preparation free of co-enzyme, would protect the zymase against the endotryptase action.

In fact it now appeared that a similar effect could be observed so that our theory regarding the nature of MEYERHOF's co-enzyme was thus further confirmed.

VI. We may now presume from the above mentioned experiments that the preparations, used by the various investigators of which the activating function on inactivated zymase is indicated by them as "co-enzyme" action, contain, dependent on the different applicated methods of inactivation, different activating principles.

On the one hand inactivation might be a consequence of withdrawal of the introductory hydrogen acceptor, on the other hand of hexosephosphate or in the end also of the antiproteolytic acting substances. Not seldom these factors will act simultaneously and this in dependence of the followed mode of operation, the nature of the used zymase preparation etc. in varying degrees. We are of the opinion that this is very much the case with the extensive investigations of H. VON EULER and his collaborators, not discussed here, but we will not go further into this now.

Thus from all the foregoing the conclusion could be drawn that it would be advisable to distinguish henceforth more co-enzymes, which might be indicated as the co-enzyme of HARDEN (introductory hydrogen acceptor) that of BUCHNER and HAEHN (hexosephosphate) and that of MEYERHOF (antiprotease).

¹⁾ TH. THOLIN, Zeitschr. f. physiol. Chemie, Bd. 115, p. 235, (1921).

We should, however, prefer to strike out the whole conception of co-enzyme from the biochemistry of alcoholic fermentation and replace it by the insight that for fermentation of the hexoses by the usual zymase preparations a certain number of factors must be realized, namely: except the presence of alkaliphosphates, that of an introductory hydrogen acceptor and of hexosephosphate¹⁾, whilst further there must be reckoned with the autolytic properties of the named preparations.

For a further documentation of this preliminary communication we refer to a thesis to be shortly forthcoming from one of us (Str.) as also to an extensive paper to be published elsewhere.

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Technical University.*

Delft, April 1927.

¹⁾ In the meantime we have strong evidence that hexosephosphate on the condition that it is present in a somewhat higher concentration, can itself fulfil the function of introductory hydrogen acceptor.

Physics. — *The thermal expansion of silver between $+101^{\circ}$ C. and -253° C.* By W. H. KEESOM and Miss A. F. J. JANSEN.
(Communication N^o. 185c from the Physical Laboratory at Leiden.)

(Communicated at the meeting of May 28, 1927).

§ 1. *Introduction.* As a continuation of our measurements of the expansion of copper ¹⁾ in the temperature range of $+101^{\circ}$ C. to -253° C. we now have investigated silver in the same temperature range.

§ 2. *Method and apparatus.* We followed the same method as described in Comm. N^o. 182a. The only change to the apparatus was that the copper inner tube was replaced by a silver one, constructed by the "Zilverfabriek" at Voorschoten. For the silveralloy is given 99.8 %.

The tube has a longitudinal seam, which is soldered with as few silver-solder as possible of a high silveralloy.

The arrangement of tube, standard meter and kathetometers was not changed.

The distance of the platinum points was determined repeatedly at room temperatures. The temperatures of the inner and outer walls were read off with mercury thermometers, graduated in tenths of a degree.

In order to reduce this results to the temperature of 0° C. for both inner and outer walls, for silver we made use of the expansion coefficient found by SCHEEL ²⁾ and for copper of our measurements (l.c.).

Besides those at room temperature, measurements were made at the boiling points of water, ethylene, oxygen and hydrogen.

In order to obtain a temperature of about 100° C., boiling distilled water was poured into the tube, and then a heating coil introduced to keep the water boiling. The temperature of the water at the top was derived from the reading of the barometer; that of the tube at the bottom by taking into account the hydrostatic pressure of the water column; the average being taken.

The temperatures of the ethylene, oxygen, and hydrogen baths were determined by means of a *Pt*-resistance thermometer, which had been calibrated several times with the helium thermometer, and which was placed half-way between the platinum points.

Before the tube was filled with a liquefied gas, it was cooled by means

¹⁾ These Proceedings 29, 786, 1926; Comm. Leiden N^o. 182a.

²⁾ K. SCHEEL, Zs. f. Physik., 5, 167, 1921.

of liquid air. This appeared to be particularly necessary in the case of hydrogen; in this case it was very difficult to get the tube filled.

§ 3. *Results.* The following results were obtained,

TABLE I.

Date	Temperature			L _{outer} —L _{inner}
	Inner tube		Outer tube	
December 18 '26	reduced to	0° C.	reduced to 0° C.	2.002 mm.
January 18 '27	"	0° "	"	2.002 "
" 22 '27	"	0° "	"	2.004 "
" 24 '27	"	0° "	"	2.002 "
February 4 '27	"	0° "	"	1.999 "
" 21 '27	"	0° "	"	2.000 "
" 24 '27	"	—182.76° "	"	4.938 "
March 10 '27	"	0° "	"	2.022 "
" 11 '27	"	0° "	"	2.015 "
" 14 '27	"	0° "	"	2.019 "
" 15 '27	"	0° "	"	2.023 "
" 17 '27	"	—102.47° "	"	3.731 "
" 22 '27	"	0° "	"	2.017 "
" 30 '27	"	—252.78° "	"	5.587 "

After the measurements in hydrogen the little point at the bottom of the inner tube appeared to be somewhat distorted, so that both points of inner- and outer tube could not be focussed at the same time. Therefore the lowest window was taken off, and this point distorted a little, so that now the distance between the points of inner and outer tube is changed. The following results were then obtained.

April 11 1927	reduced to	0° C.	reduced to 0° C.	5.127 mm.
" 12 1927	"	0° "	"	5.128 "
" 26 1927	"	0° "	"	5.128 "
May 2 1927	"	+101.19° "	"	3.295 "
" 5 1927	"	+101.10° "	"	3.301 "
" 6 1927	"	0° "	"	5.128 "

From this the following changes of length were derived :

Temperature range	0° C. to +101.10° C.	expansion	1.827 mm.
"	" 0° C. " -102.47° C.	contraction	1.712 mm.
"	-102.47° C. " -182.76° C.	"	1.207 mm.
"	-182.76° C. " -252.78° C.	"	0.649 mm.

Length of the inner tube (from point to point) = 945.30 mm.

The change of length as a function of the temperature is represented in figure 1.

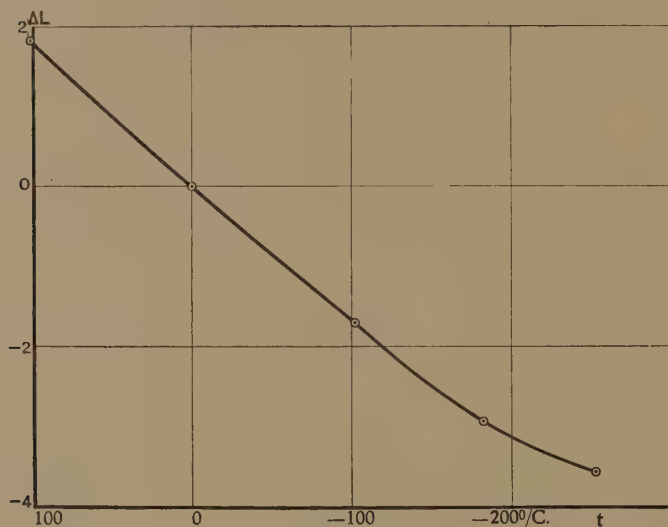


Fig. 1.

§ 4. *The expansion coefficient.* From the results the following mean expansion coefficients were calculated.

Temperature range	$\bar{\alpha} \times 10^6$
+101.10° C. to 0° C.	19.15
0° " " -102.47° "	17.61
-102.47° " " -182.76° "	15.89
-182.76° " " -252.78° "	9.81

In Fig. 2 we derived graphically $\frac{1}{l} \frac{dl}{dt}$ by drawing a smooth curve, so

that, for the temperature ranges mentioned above,

$$\int_{t_1}^{t_2} a \, dt = \bar{a} (t_2 - t_1).$$

The equality of the areas in question was tested with the aid of a planimeter.

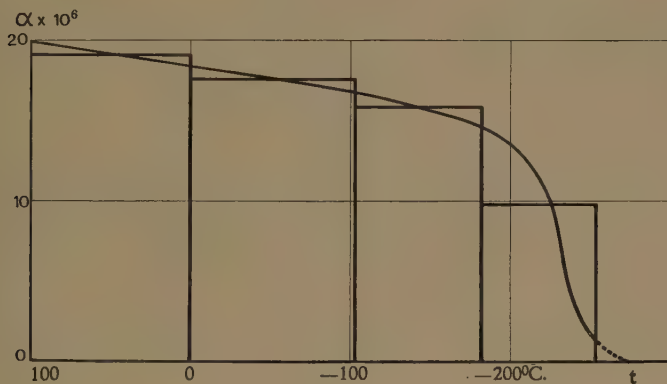


Fig. 2.

§ 5. *Representation in a formula.* In analogy to the formula for the expansion of copper, a fourth degree equation was made of the form :

$$L_t = L_0 \left[1 + \left\{ a \left(\frac{t}{100} \right) + b \left(\frac{t}{100} \right)^2 + c \left(\frac{t}{100} \right)^3 + d \left(\frac{t}{100} \right)^4 \right\} 10^{-6} \right].$$

The coefficients were calculated from the observations ; the following values were obtained :

$$a = 1779.066$$

$$b = 34.641$$

$$c = 59.343$$

$$d = 35.754$$

§ 6. *Comparison with previous observations.* Previous investigations on the expansion of silver below 100° C. were made by FIZEAU ¹⁾ in the temperature range from 0° C. to 100° C., AYRES ²⁾ in the temperature range from +99° 3 C. to -184° C., SIMPSON ³⁾ in the temperature range from

¹⁾ H. FIZEAU, C.R. 68, 1125, 1869.

²⁾ H. D. AYRES, Phys. Rev., 20, 38, 1905.

³⁾ Cf. J. S. SHEARER, Phys. Rev., 20, 52, 1905.

0° C. to -180° C., DORSEY ¹⁾ in the temperature range from +10° C. to -180° C., HENNING ²⁾ in the temperature range from +16° C. to -191° C., SCHEEL ³⁾ in the temperature range from 0° C. to 500° C., LINDEMANN ⁴⁾ in the temperature range from +19° C. to -253° C.

The measurements of LINDEMANN do not allow a direct comparison as they are relative to fused quartz, of which the expansion coefficient is not known.

For the other observers we have calculated from their results the mean expansion coefficients for the following temperature ranges, and compared these coefficients with ours in table II.

TABLE II.

$\bar{\alpha} \times 10^6$					
temp. range observers	0° C. to + 100° C.	0° C. to - 100° C.	0° C. to - 180° C.	- 100° C. to - 180° C.	- 180° C. to - 250° C.
FIZEAU	19.36				
AYRES	18.90	18.80	17.67	16.10	
SIMPSON		18.82	17.78	16.47	
DORSEY		18.43	17.24	15.74	
HENNING			17.05		
SCHEEL	19.68				
KEESOM and Miss JANSEN	19.14	17.63	16.89	15.94	10.37

¹⁾ H. G. DORSEY, Phys. Rev., 25, 88, 1907.

²⁾ F. HENNING, Ann. d. Phys., 22, 631, 1907.

³⁾ K. SCHEEL, Zs. f. Phys., 5, 167, 1921.

⁴⁾ CH. L. LINDEMANN, Physik. Zs., 12, 1197, 1911.

Zoology. — *The Nemas Anchylostoma and Necator*. II. *New researches on the larvae of Anchylostoma duodenale* DUB. and *Necator americanus* STILES. By J. H. SCHUURMANS STEKHOVEN Jr. (Communicated by Prof. W. SCHUEFFNER).

(Communicated at the meeting of April 30, 1927).

Sooner than expected, thanks to the kindness of Prof. W. SCHUEFFNER, Director of the Institute for Tropical Hygiene at Amsterdam, I was afforded another opportunity to continue my studies on hookwormlarvae. I am glad to acknowledge here with thanks Prof. SCHUEFFNER for his valuable help, which facilitated my work very much.

On the 26th of February he mailed me 3 samples of stools, produced by Javanese, working on board of one of the ships of the Steamship Cie "Nederland". All stools contained a good quantity of hookwormeggs: moreover eggs of *Ascaris* and *Trichocephalus* were present.

On the 28th of the same month, that is, immediately after the receipt of the samples, I prepared my cultures by mixing a small quantity of the stools with sterilised soil. These I placed in an incubator at 27° C. After the appearance of the first larvae I daily made a series of observations on living hookwormlarvae. For that purpose I isolated them in a drop of water on a slide, covered the drop with a circular coverglass and sealed the whole with wax of a taper, making thus a temporary mount. In the case the larvae made vehement movements, I warmed the slide cautiously above a flame, till the larvae came into a condition of "Wärmestarre".

With this research I aimed: firstly to give a correct, practical diagnose of the third stage, infectious larva of *Anchylostoma duodenale*, and secondly to complete my observations upon the first stage and second stage larvae of *Necator americanus* and possibly also of *A. duodenale*.

Since 90 % of all hookworms, which Javanese harbour, belong to the Genus *Necator*, I could be quite sure to encounter larvae of the latter Genus. So I found indeed larvae of *Necator* in all cases, whereas one of the amples only produced a sufficient quantity of larvae of *A. duodenale*.

1. *The third stage larva of A. duodenale* DUB.

"We could divide the hookwormlarvae, cultivated from human stools—where eventually both species of hookwormlarvae may be present, although the chance is greater, at least at Weltevreden, that we have to do with larvae of *Necator americanus* — into two groups. In the larvae, belonging to the first group, the hooks at the distal end of the

cylinder run parallel (Fig. 7); the same structures enclose a vase-shaped cavity in the other group of larvae (Fig. 9). In the majority of the cases these rods with their hooklets did run parallel, as is also true for larvae, derived from pure cultures of *Necator americanus*. Almost all larvae, cultivated from canine stools presented cylinders with diverging hooklets. No parallelity was found", wrote my wife and I as a result of our researches at Weltevreden in 1921 (published in 1923).

At that moment we were not far from the final solution of the problem of diagnosis of hookwormlarvae. Yet the collected data were not sufficient to draw decisive conclusions from our observations. My studies in Washington allowed me to fill out the gaps.

If the word "almost" is dashed out from the cited sentence: "almost all larvae etc.", it is clear, that the observations, which we both made in 1921 were quite correct and that the cited alinea points to one of the most important differences, which exists between the larvae of the Genera *Anchylostoma* and *Necator*.

Our expression "gespreide haakjes", "diverging hooklets" was misunderstood by VAN THIEL. If he had confronted the latter expression with the indication "sikkelvormige haakjes", "falciform hooklets", which we used on page 11 of the mentioned paper for *Anchylostoma* larvae, he would not have misunderstood what we meant with the word "gespreid".

If the figures, accompanying this paper, are compared minutely with those, printed in the cited paper of my wife and me, it will be quite clear, that the median sides of the rods with their hooks run parallel in the larvae of *Necator*, whereas in all *Anchylostoma* species, known thus far, the hooklets or apical clasps diverge.

The head of the third stage larva (Textfig. 1; Pl. I) possesses, just the same as that of *A. caninum* 3 lips (l.), behind which the bodywall has been constricted slightly (Pl. I, Fig. 2 c). LOOSS speaks about "a simple ring-shaped lip which is marked off from the rest of the body by a slight annular constriction not always clearly visible (Pag. 349)", when he treats the first stage larva of the same species. PERRONCITO and ALESSANDRINI however observed apparently the lips, since the former speaks of a "testa triloba".

These lips are, just as in *A. caninum*, more transparent than the next plasmatic part of the head. Therefore VAN THIEL's microphotogram 2 (1924) depicts them as a clear space. LOOSS gives in his monograph (Pl. XIV, Fig. 204) a schematical figure of the head of *A. duodenale* seen on top; here 6 papillae encircle the oral opening, whereas the circumference of the head is broken at two opposite places by a couple of slanting lines. Possibly the latter indicate the amphids, although LOOSS denies the existence of cervical organs opposite to PERRONCITO. If this supposition holds true, a controverse arises between LOOSS's observations and mine, since the former found 6 papillae, whereas I found 4 papillae only, when the head was seen on top. Lack of time forced me to

postpone the making of headcuttings of larvae of *A. duodenale*. I think however I may safely assume that both species of *Anchylostoma* larvae have the same number (4-) of labial papillae. In profile both subdorsal papillae present one papilla only.

It is interesting to note, that LOOSS, although he denied the existence of the various organs at the head of *A. duodenale*, apparently has seen them. Pl. XVI, Fig. 205, 207 and probably also Fig. 202 depicts the amphids together with the nerve fibres, which run from here in caudal direction. Confer also Fig. 199. STEINER came to the same conclusion. The amphids, their pipe-shaped amphidial pouches, which contain the threadlike terminals, as also the widened parts, which come next, are likewise to be found on VAN THIEL's microphotogram 2 (1924) of a larva of *A. caninum*, although he does not mention these organs in his text.

In *A. duodenale* the amphids (*amph.*) have a similar structure. Their aperture (*amph. o.*) is elliptical and wide; just behind the constriction of the head they communicate with the exterior. The amphidial pouch (*amph. p.*) is rather wide and contains 2 terminals (Pl. I, Fig. 5); the next part of this sense-organ, separated from the former by a constriction, widens somewhat more (*amph. n. p.*) and stretches along the esophagus till it meets with the nerve ring (*n. r.*). Here again it is

Textfigure 1.

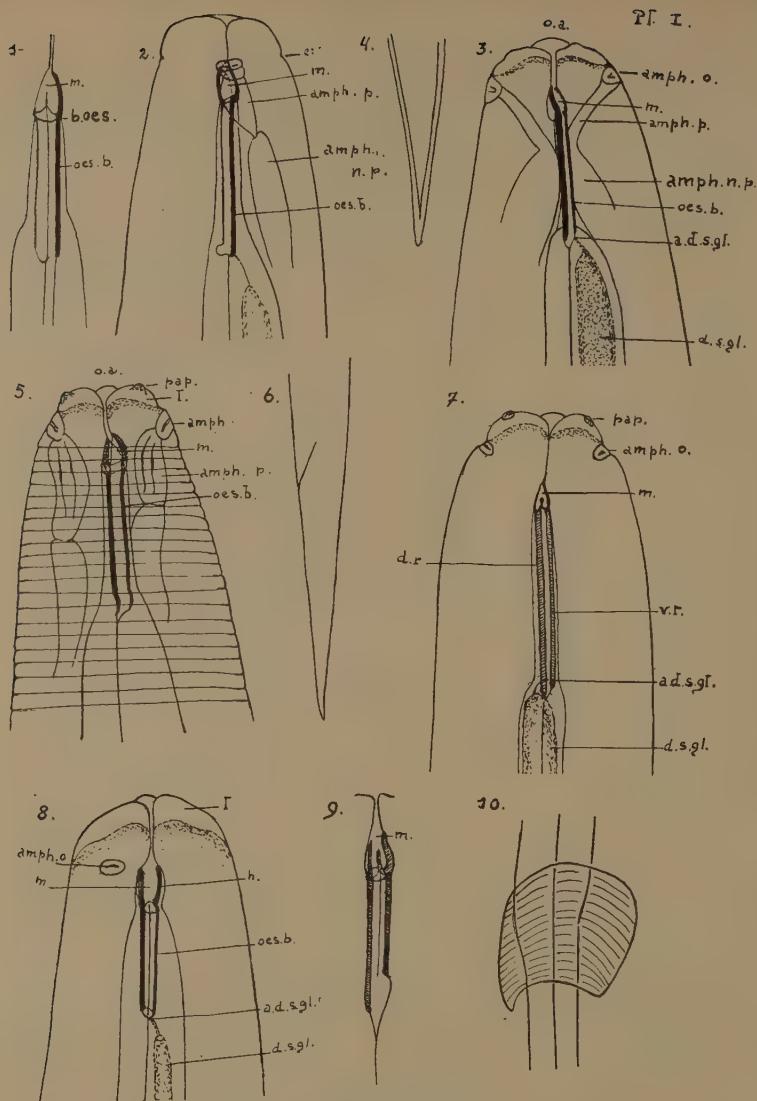
Anchylostoma duodenale, third stage larva.

Fig. 1. The headend, Enlarg. ÖI Imm. 1.25 mm., Oc. 12, Tubus 140 mm. $\times \frac{2}{5}$.

Fig. 2. Tailend of third stage larva, Enlarg. ÖI Imm. 1.25 mm. Oc. 1, Tubus 140 mm. $\times \frac{2}{5}$.

a. d. s. gl. = aperture of dorsal salivary glands; *ant. h.* = anterior horns of excretory apparatus; *a. v. s. g.* = aperture of subventral salivary glands; *amph.* = amphid; *amph. p.* = amphidial pouch; *b. oes.* = beginning of esophagus; *exc. c.* = excretory cell; *exc. c. p.* = porus of excretory cell; *l.* = lip; *m.* = oral cavity; *n. r.* = nerve ring; *oes. b.* = esophageal rods; *oes. bu.* = bulbous esophagi; *pap.* = papilla; *post. h.* = posterior horns of excretory apparatus; *sal. gl.* = salivary glands; *sph.* = sphincter; *str. oes.* = constriction of esophagus; *v. s. gl.* = ventral salivary glands.

Textfigure 1.



Anchylostoma duodenale, third stage larva.

Fig. 1. Esophagus cylinder, Enlarg. Ö Imm.

1.25 mm., Oc. 12, Tubus 140 mm. $\times \frac{2}{3}$.

Fig. 2. Head, profile, Enlarg. as in 1.

Fig. 3. Head, dorsolateral, as in 1.

Fig. 4. Tail of third stage larva, as in 1.

Fig. 5. Head, dorsal view, Enlarg. Ö Imm.

1.25 mm., Oc. 18, Tubus 140 mm. $\times \frac{2}{3}$.

Fig. 6. Tail, third stage larva, Enlarg. Ö Imm.

1.25 mm., Oc. 1, Tubus 140 mm. $\times \frac{2}{3}$.

Fig. 7. Head, dorsal, as in 1.

Fig. 8. Head, lateral, as in 1.

Fig. 9. Esophagus cylinder, as in 1.

Fig. 10. Nervering, as in 1.

a. d. s. gl. = aperture of dorsal salivary gland; amph. o. = amphidial opening; amph. p. = amphidial pouch; amph. n. p. = amphidial nerve part; b. oes. = beginning of esophagus; c. = constriction; d. s. gl. = dorsal salivary gland; d. r. = dorsal rod; h. = hooklet; l. = lip; m. = oral cavity; o. a. = oral aperture; oes. b. = rodshaped thickenings of the esophagus cylinder; pap. = papilla; v. r. = ventral rod.

obvious that the amphidial pouches are placed at right angles to each other; the proximal part of the amphids however runs parallel with the intestine (Compare also *A. caninum* l. c. Pl. I, Fig. 2 and 3). Now it is clear that the shape and structure of the amphids in *Necator* deviates distinctly from the aforementioned scheme.

The oral cavity (*m.*) and the distal portion of the esophagus cylinder distinctly differ from what I found in *A. caninum*. The entrance of the oral cavity is short and cylindrical, but soon it widens and assumes the shape of a vase. A circular aperture at the basal end of the vase opens into a cylindrical tube, which is lined with parallel running rods (*oes. b.*). The dorsal rod (*d. r.*) is thicker than the ventral one (Pl. I, Fig. 7 *v. r.*) which moreover is slightly shorter than the former, whereas the third rod (Confer *A. caninum*) is reduced to a fine cuticularised line. Both dorsal and ventral rods are prolonged at their proximal ends into curved clasps (Pl. I, Fig. 8, *h.*) which double the curvature of the vase; each clasp, which is almost as strong as the hooklets in *Necator*, is connected with the fine cuticularised third line, whereas fine lines connect the top of each clasp with the base of its antagonist (Compare Pl. I, Fig. 1, 2, 3, 5, 7, 9). Where rod and clasp connect, the cavity is bordered by a ring-shaped thickening of its wall.

It is of importance I think to mention the fact that both clasps are not placed in the same plane, but are found in planes which stand at right angles to each other (Pl. I, Fig. 3).

At its caudal end the cuticularised esophageal cylinder, (the rods ought to be considered as thickenings of the wall of the latter) is closed by an irregular ring.

In rare instances, but in dead larvae only, the clasps protrude from the oral opening. SVENSSON and KESSEL pointed to the circumstance that in *A. caninum* "the difference in density of the dorsal and ventral lines is less pronounced than in *A. duodenale*", which observation I could affirm. Their Figure 8 may serve to prove the aforementioned fact, that rods and hooks show more resemblance with the same structures in *Necator* than with those in *A. caninum*. In this figure one of the hooks is depicted as a dotted line, which proves that these authors have observed the difference in situation of the clasps I described above.

The esophagus is attached to the ring, found at the base of the oral cavity (Pl. I, Fig. 1). Its outer wall runs at first parallel with the described rods, but afterwards it widens just behind the proximal ends of these cuticularisations. At this point (*a. d. s. gl.*). (Compare also LOOSS: Fig. 196, 197a, 202, 205) the excreta of the dorsal salivary gland are poured out into the lumen of the esophagus.

The dorsal salivary gland (*d. s. gl.*) is voluminous and may be traced easily along its whole course. The subventral salivary glands (*v. s. gl.*) are likewise very obvious. At the proximal end of the anterior portion of the esophagus, just in front of the nerving, these glands open into

the esophageal lumen with a wide ampulla (Textfig. 1, *a. v. s. gl.*), just as in *Necator*. Therefore the salivary glands of *A. duodenale* show a greater resemblance to the corresponding organs of *Necator* than to the same of *A. caninum*. This resemblance becomes still greater if we compare the glandular portion of the bulbus esophagi of *A. duodenale* with that of *Necator*. PERRONCITO gives a clear picture of the same (Fig. 24). This figure answers better to the reality than the figures of LOOSS, in which the glandular parts are depicted as clear vacuoles. As in *Necator*, *A. duodenale* presents lobular, strongly constricted salivary glands (Textfig. 1, *s. gl.*). These constrictions divide the bulbus esophagi into 3 distinct pale rings. I never observed in *A. duodenale* a vacuolised middle portion as was found in *A. caninum*. The narrowed part of the esophagus gradually widens to the bulbus esophagi, just as in *A. caninum*. The sphincter (*sph.*) is in shape and in the mode of junction with the esophagus quite similar to the same structure in *A. caninum*. The only difference with the latter consists in the fact that here this part is more granular and therefore does contrast less with the bulbus. The lumen of the bulbus widens towards the sphincter (Textfig. 1, *sph.*); at this level the wall of the lumen is thickened at two opposite places. It is not always easy to observe this fact, since the strongly granular glands prohibit to see it through. Similar, although less pronounced thickenings I found in *A. caninum*. It may be thought possible that they do represent the remainder of a valvular apparatus, such as is found in first and second stage larvae, but I doubt, that it still functions as such.

LOOSS describes it on page 359: "The valves of the bulb are not actually reduced but only elongated and flattened; at the same time they loose their glistening chitinous covering, so that very close inspection is needed in order to recognize them in their altered shape".

The excretory apparatus of *A. duodenale* is not so distinctly visible as in *A. caninum*, but shape and structure are identical. The middle portion of the excretory cell (*exc. c.*) is swollen, the posterior horns (Textfig. 1, *post. h.*) are very slender, whereas the anterior horns (*ant. h. e. a.*) are short and wide.

The tail of the third stage larva (Pl. I, Fig. 4, 6) tapers, just as in *A. caninum*. The tail of the sheath is very long, finely pointed (Textfig. 1, 2) and resembles therefore the same part in *Necator*; so LOOSS describes the tail of the third stage larva correctly as strongly shortened in comparison with the second stage larva. The skin is finely ringed (Pl. I, Fig. 5); the distance between two rings is $0.9\ \mu$. Finally some dimensions may be mentioned. Now it appears that *Anchylostoma* larvae are somewhat longer on the average than the corresponding larvae of *Necator*. If we have to do with a mixture of both larvae, the *Anchylostoma* larvae are generally the longer ones. Again I compared the data collected by my wife and me with the dimensions of newly measured larvae. As our knowledge of the factors controlling growth is

quite insufficient, it is impossible to draw any definite conclusions from these data. Here again it was clear that *Necator* is somewhat shorter on the average than *Anchylostoma*, but at the other hand both curves overlap, which fact lessens the value of similar data.

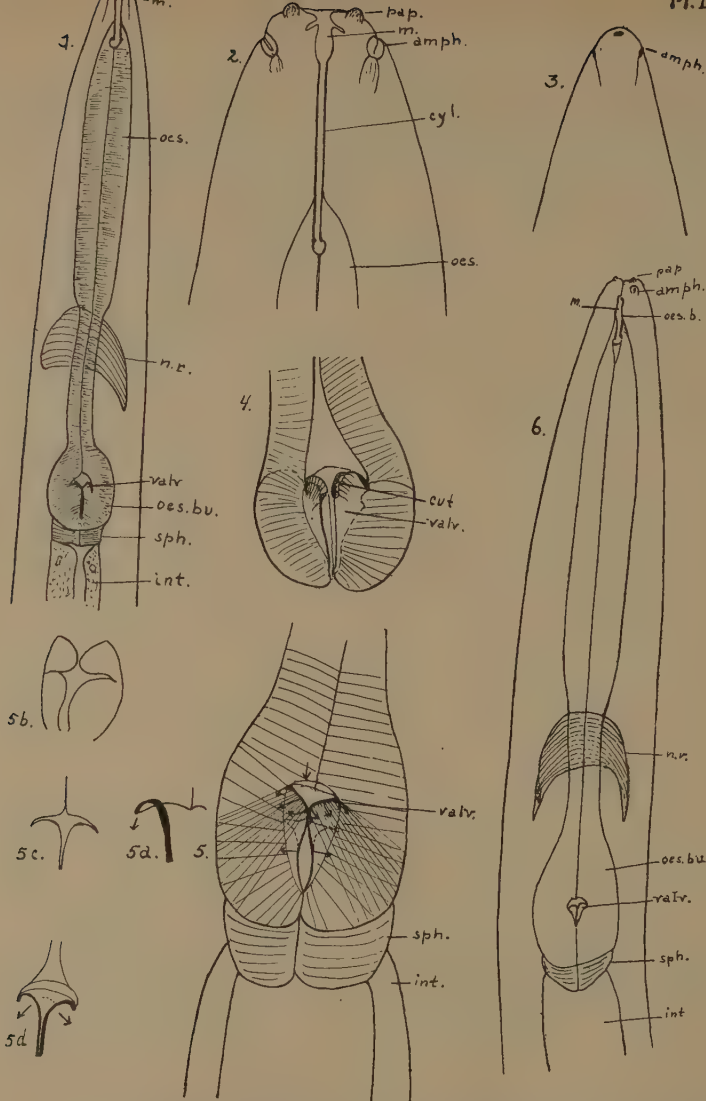
Dimensions of larvae	A. duodenale			Necator americanus		
	Age	3rd stage	2nd stage	Age	3rd stage	2nd stage
Total length Utrecht	11 d.	608—627 μ	—	1 d.	—	—
" " Batavia	5—8 d.	384—624 μ	—	5—8 d.	399—642 μ	—
" " Utrecht	1 d.	—	665—684 μ	1 d.	—	532—634 μ
" " Batavia	5—8 d.	—	471—654 μ	5—8 d.	—	462—666 μ
Head to end esoph. Batavia and Utrecht	5—11 d.	135—165 μ	—	5—11 d.	102—177 μ	—
Tail = anus till point, Batavia	5—8 d.	36—75 μ	—	5—8 d.	38—90 μ	—
" Utrecht	11 d.	62—78 μ	—	—	—	—
" "	11 d.	—	138 μ	11 d.	—	—
" Batavia	5—8 d.	—	69—141 μ	5—8 d.	—	81—150 μ

In length of tail and sheath both species agree almost wholly; comparatively the tail of *A. duodenale* is even slightly longer than that of *Necator americanus*, a fact predicted by VAN THIEL.

As a summary it is clear that the third stage larvae of *A. duodenale* have many points in common with *A. caninum*, but differ i.a. in the density of the crescent-shaped clasps, in structure and volumen of the salivary glands and their efferent ducts and in the length of the tail of the second stage larva (sheath). In these features the third stage larvae of *A. duodenale* resemble the corresponding larvae of *Necator americanus*.

2. The first stage larva of *Necator americanus*.

Under the present circumstances it was impossible for me to make pure cultures, starting from uteruseggs of each of the species of nemas, which I wished to diagnose. Since I was able to identify the larvae of my cultures after they had reached the infectious stage and one of my cultures yielded third stage larvae of *Necator americanus* only, I could be almost sure that the younger larvae, found in the same culture, ought to be reckoned to this species. Now I will describe here the



Necator americanus, first and second stage larva.

- Fig. 1. *Necator*, first larva, Enlarg. Öl Imm. 1.25 mm., Oc. 6, Tubus 140 mm. $\times 2/3$.
 Fig. 2. Head of same larva, Enlarg. Öl Imm. 1.25 mm. Oc. 18, Tubus 140 mm. $\times 2/3$.
 Fig. 3. First stage larva, empty sheath.
 Fig. 4. Second stage larva, bulbus esophagi Enlarg., Öl Imm. 1.25 mm., Oc. 12, Tubus 140 mm. $\times 2/3$.
 Fig. 5. First stage larva, bulbus esophagi, same animal as Fig. 1, Enlarg. as in Fig. 2.
 Fig. 5a, b, c, d, different stages of the valvulae esophagi, while functioning.
 Fig. 6. Second stage larva, head portion of a larva, Enl. as in Fig. 1.

amph. = amphid; *cyl.* = esophagus cylinder; *cut.* = cuticularisation; *int.* = intestine; *m.* = oral cavity; *n.r.* = nerve ring; *o.a.* = oral aperture; *oes.* = esophagus; *oes. b.* = beginning of esophagus; *oes. bu.* = bulbus esophagi; *pap.* = papilla; *sph.* = sphincter; *valv.* = valvula. The arrows in Fig. 5 indicate the direction of the movements; the dotted lines show the shape of the lumen, on the moment the valvulae are open, see also Fig. 5b.

younger larvae of *Necator* only, although I made some observations upon the younger larvae of *A. duodenale*. I will however postpone the publication of the data I collected upon the younger larvae of this *Anchylostoma*-species, till I have had another opportunity to study them more minutely. As a matter of fact my notices upon the younger larvae of *Necator* are also more or less fragmentary, since the first and second stage larva accomplishes its development in a few days. Four or five days after the preparing of the cultures ensheathed third stage larvae usually are present.

The first stage larva of *Necator americanus*, $285\ \mu$ long is in different respects less developed than the third stage larva; esophageal glands are not yet visible and no excretory apparatus has to be found, although LOOSS mentions a group of cells, which forms the matrix of this organ. From a diagnostical point of view the head and its organs are the most important. The anterior head end (Pl. II, Fig. 1, 2) is rounded; no lips are present. Amphids (*amph.*) are to be found and the labial papillae (*pap.*) are especially conspicuous when the larva is observed under sufficient magnification. The more or less cylindrical oral cavity (*m.*) is short and opens immediately into the esophageal cylinder (*cyl.*). The cuticularisation of its wall presents no special differentiations but is wholly smooth. Proximally this cylinder ends with a ring. Here the parallel-sided anterior portion of the esophagus sets off. Where one finds the nerve ring the esophagus is constricted over a remarkable distance, afterwards it swells rather suddenly to the bulbus esophagi (*oes. bu.*). In this way the esophagus is divided into 3 well separated regions (Pl. II, Fig. 1). These divisions fade away during growth and after the ecdyses, but never disappear wholly. The sphincter (*sph.*) situated between intestine and esophagus, does not protrude so wart-like into the lumen of the latter as this is the case in the third stage larva.

The intestine (*int.*) possesses a comparatively thick wall, whereas its lumen is narrow and sinuous, till it opens, just in front of the long tapering tail (Pl. III, Fig. I), through the anus (*a.a.*). The rectum can be closed by the sphincter ani (*sph. a.*), which has a fine boring. The peristaltic movements, which the intestine showed during digestion were easily to follow; the lumen of the intestine alternately widened and narrowed and small particles were expelled by the anus.

It is necessary to pay more attention to the valvular apparatus, situated in the bulbus esophagi (*valv.* Pl. II, Fig. 1). The apical wall of the lumen bulbi is not strengthened by cuticularisations and resembles a mediastine (Pl. II, Fig. 5 *a—d*); it vaults the caudal part, which has a triangular optical section, although in profile two parts only are visible. The proper valvulae form the distal portion of the latter part; their upper borders are strongly thickened at their median angles (Pl. II, Fig. 4, 5, 5a); the walls bordering the lumen in caudal direction are likewise strongly cuticularised (Pl. II, Fig. 1, 4). In optical longitudinal section this part

gets therefore the shape of an Y. At the corners, where the branches of the Y pass into the longitudinal leg, the thickenings are strongest. From here and from the slanting branches muscles radiate (Pl. II, Fig. 4, 5). Their function is to open the valvulae and by doing this to widen the lumen of the esophagus which gets then its sucking power.

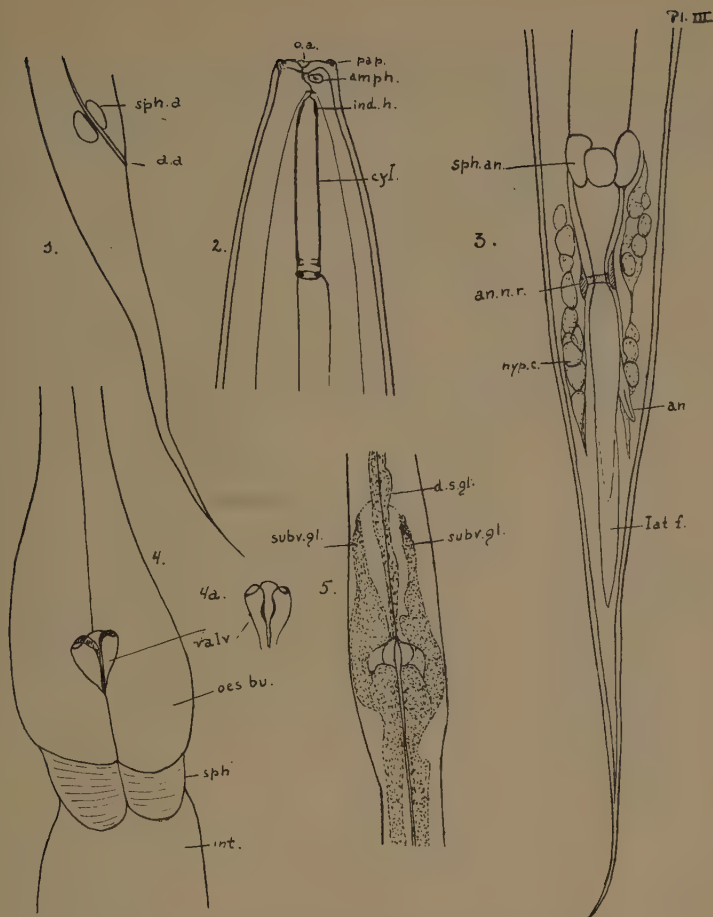
During the ingestion of food particles, the latter are driven forward by means of peristalsis. Then they reach the small space, just in front of the Y which I will call the atrium bulbi. Now the muscles which radiate from the points, where the atrium vault articulates with the Y, contract. I imagine that things happen in this way that the inflow of the fluid in the atrium affords the stimulus to which the muscles answer by contraction. Now the vault of the atrium flattens and functions therefore as the sucker of a pump, which drives the fluid forward in caudal direction and throws it against the valves. The described radiating muscles serve two purposes. The point where the vault articulates with the branches of the Y is torn backwards and at the same time do the branches of the Y turn round these points as fixed axes in outward direction, whereas the leg of the Y placed in the longitudinal axis of the body remains almost in the same position.

Since the thickenings of the wall are not evenly distributed and therefore the elasticity is not the same at all points, a complicate vase-shaped figure is formed (Pl. II, Fig. 5b). As soon as the contraction of the muscles ceases and the vault regains its former position, the valvulae come to rest again, thanks to the elasticity of the cuticularisations (*cut.*); now the upper borders of the thickenings overlap. After the valvulae have closed off the atrium from the rest of the bulbus, the food materials are driven forward through the caudal portion of the bulbus by peristalsis.

Several authors described the valvular apparatus of the bulbus esophagi, most of them for *A. duodenale*, but I think they have not emphasized enough the complexity of its structure.

PERRONCITO mentions "le bulbe pharyngien, muni de dents chitineuses, renfermant dans sa partie centrale un espace triangulaire". LOOSS is in opposition to PERRONCITO, which asserts that the bulbus is provided with teeth. He also disputes with LEICHTENSTERN who considers the bulbus to be "kein Zahn-oder Kau-Apparat, sondern eine Pump- oder Ansaug-Vorrichtung". LOOSS thinks that the "teeth represent the primordia of the future esophageal valves of the adult worm and constitute an apparatus for preventing regurgitation. In order to secure an inflow of fresh nourishment through the mouth during such an extension, the opposite end of the oesophagus must be closed, as will be understood without further explanation. The teeth under discussion provide for this closure and thus represent valves. They are closed in a state of rest and become opened by the contraction of the muscle fibres proceeding from them, while when the muscles relax, they spring back into this former position".

The description I gave makes it sufficient clear, I think, that I join



Necator americanus, first, second and third stage larva.

Fig. 1. Tail of first stage larva, same animal as Pl. II, Fig. 1, Enlarg. same.

Fig. 2. Head of second stage larva. Enlarg. ÖI Imm. 1.25 mm. Oc. 12, Tubus 140 mm. $\times \frac{2}{3}$.

Fig. 3. Tail of second stage larva of *A. duodenale* of *N. americanus*? Enlarg. ÖI Imm. 1.25 mm. Oc. 6, Tubus 140 mm. $\times \frac{2}{3}$.

Fig. 4. Bulbus esophagi second stage larva *Necator*, Enlarg. as 2.

Fig. 4a. Valvulae bulbi esophagi.

Fig. 5. Third stage larva *Necator*, salivary glands. Enlarg. as in 2.

a. a. = anal aperture; an. n. r. = anal nerve ring; an. = anus; amph. = amphid; cyl. = esophageal cylinder; d. s. gl. = dorsal salivary glands; hyp. c. = hypodermis cells; ind. h. = indications of clasps; int. = intestine; lat. f. = lateral cords; m. = oral cavity; oes. bu. = bulbus oesophagi; sph. = sphincter; sph. a. or sph. an. = sphincter ani; subv. gl. = sub-ventral salivary glands; valv. = valvulae.

LEICHTENSTERN's opinion, since the atrial vault (Pl. II, Fig. 5*b—d*) prohibits a regurgitation. Now I will abstain from a discussion over the value which the valvular apparatuses of free living and parasitic nemas have in respect with phylogenetical speculations. I think however that the loss of the valvular apparatus during the lifecycle or even the loss of its function speaks in favour of a secondary reduction, probably in relation with parasitism and yields another argument for STEINER's opinion that parasitic nemas are not primitive forms.

3. *The second stage larva of Necator americanus.*

The second stage larva resembles in many respects the first stage larva; from the third stage larva it may be distinguished at once by its much longer tail. The head is more *Necator*-like than the head of the first stage larva of the same species; its anterior end is flattened in the middle (Pl. III, Fig. 2; Pl. III, Fig. 6); lips are absent, the papillae (*pap.*) are to be seen as glistening warts; the amphids (*amph.*) are situated next to the oral opening; oral cavity short, cup-shaped. The covering of the esophageal cylinder (*cyl.*) has undergone slight changes only; thickenings appear as indications of the later hooklets (Pl. III, Fig. 2, *ind. h.*), which resemble in this species the pollensacs of grasses; at the proximal end of the cylinder there is a couple of rings, whereas further on, at the beginning of the caudal portion of the esophagus 2 strongly refractive points are very obvious. The ring at the proximal end of the cylinder seems to be thickened remarkably. The indications of the hooks do not protrude into the oral cavity as is the case in third stage larvae of *Anchylostoma* (Compare LOOSS, page 349). The bulbus esophagi (*oes. bu.*) contains the valvulae (*valv.*) I described for the first stage larva. Pl. III, Fig. 4 shows beautifully how the mentioned muscles radiate from the thickenings of the Y-branches. Salivary glands are faintly visible just now; the bulbus esophagi presents three regions as traces of the later lobes. The nerve ring (Pl. II, Fig. 6 *n. r.*) moved nearer to the bulbus esophagi than in the first stage larva. The excretory apparatus is present but does not yet function. The sphincter (*sph.*) protrudes wartlike into the lumen of the intestine (*int.*).

Finally some fragmentary data may be added. Pl. III, Fig. 3 shows an empty skin, which represents I think the first ecdysis; the amphids, of which the apical portion apparently has been stripped off, are plainly visible.

Further Pl. III, Fig. 3 depicts the tail of a second stage larva probably of *Necator americanus*, but as this larva was isolated from a culture, which contained a mixture of larvae of both human species of hookworm larvae, I could not make out this point. The cited figure depicts the sphincter ani (*sph. an.*), the anal nerve ring (*an. n. r.*), the anus (*an.*) and the cords of hypodermis cells (*hyp. c.*) which divide the cylindrical

body into fields. The bulbus esophagi of the same animal (Pl. III, Fig. 4, 4a) contained valvulae; the cuticularised esophageal cilinder, situated next to the oral cavity, had the same shape as in the aforementioned larvae. Bulbar valvulae I found in all larvae younger than the third stage and so they must be also present in both first and second stage larvae of *A. duodenale*.

My description of the third stage larva of *Necator americanus* needs a correction. There I spoke of a rudimentary anterior bulbus and depicted in Textfig. A of my former paper on hookworm larvae a special muscularisation at this point. In the material which served as a base for the present paper I could not find again a similar muscularisation; the lumen of the esophagus was slightly widened only at the level of the ampulla of the subventral salivary glands (Compare Pl. III, Fig. 5).

Utrecht, March 28th, 1927.

POSTSCRIPTUM.

Professor SCHUEFFNER drew my attention to a lecture of VAN THIEL, delivered by the latter author on an assembling of the "Vereeniging voor Tropische Geneeskunde" on March 13th, during which VAN THIEL declared that his communications on the larvae of *A. caninum* were in reality observations on *Uncinaria stenocephala* larvae. In the mean time I examined again the worms, expelled from the same dog, whose stool yielded a large quantity of *A. caninum* larvae when it served for culture purposes, and again I found specimens of *A. caninum* only.

May 24th, 1927.

REFERENCES.

SCHUURMANS STEKHOVEN, J. H. Jr. en SCHUURMANS STEKHOVEN—MEYER, A. W., Voorbereidende onderzoekingen met betrekking tot de differentiatie van de larven van de mijnwormen van mensch en hond, Tijdschr. v. Vergelijk. Geneesk., Dl. 10, p. 1, 1923.

See for other papers also:

SCHUURMANS STEKHOVEN, J. H. Jr. The Nemas *Anchylostoma* and *Necator* I. These Proceedings Vol. 30, No. 1, p. 113, 1927.

STEINER, G. Some Nemas from the alimentary tract of the Carolina Tree Frog (*Hyla carolinensis* PENNANT) with a discussion on some general problems of Nematology, Journal of Parasitology Vol. 11, p. 1—32, 1924.

STEINER, G. The problem of Host selection and Host specialisation of certain plant-infesting Nemas and its application in the study of nemic pests, Phytopathology, Vol. 15, p. 499—534, 1925.

Histology. — *Concerning an Optic Phenomenon in Tendons.* (Part. III).
by Prof. G. C. HERINGA M. D. and M. MINNAERT SC. D. (Communicated by Prof. J. BOEKE).

(Communicated at the meeting of April 30, 1927).

In two preceding numbers of these Proceedings ¹⁾ we have pointed to polarized interference-phenomena, produced in thin longitudinal sections of a cow's tendon. We have demonstrated that these phenomena can be completely explained by assuming that the fibres of the tendon behave like long, quasi-uniaxial crystals, and by conceiving them as not running in a straight line but 1°. either in the shape of sine curves in a plane, or 2°. in the shape of helices. Various arguments have been brought forward for adopting the latter hypothesis. We are now in a position to furnish conclusive evidence for this hypothesis: we found namely, *that the cow's tendon rotates the polarized light excessively*. This can only be explained on the basis of the helix-hypothesis. It will appear that the amount of rotation can be computed beforehand, and that the order of magnitude is confirmed by the observation. In order to be able to observe the rotation, we had to look in the direction of the fibres of the tendon, so that we had to make transverse sections. These were taken as thick as possible in order to get a strong rotation; however, the drab colour and the inevitable irregularities inside the tendon did not allow of greater thickness than 3 mm. The preparations were derived from the toe-tensor of the cow. They were cut unfixed by means of the freezing microtome as smoothly as possible, and embedded in levulose-gelatin. It is owing to this means of embedding that the preparations of 3 mm. thickness were still sufficiently translucent.

For our observations we used a simple polarization apparatus of NÖRREMBERG with the parallel light of a mercury lamp, from which the blue and the violet radiations had been removed by means of a filter of potassium-bichromate; the active wavelength was computed as 560 $\mu\mu$.

The tendon-preparations were placed over the opening of a diaphragm, which shut off all the side-light, and they were viewed between crossed polarizers. We shall now first describe the phenomena, which were exhibited by our finest preparation, and which approximated most closely to what we anticipated theoretically.

This tendon-preparation appeared to be extremely dextro-rotatory; the levulose itself, at the side of the preparation, yielded a feeble rota-

¹⁾ These Proceedings 35, p. 939.

tion in the opposite sense. The angle between the two adjustments amounted to about 30° (on an average for two observers), which we shall consider as the particular rotation of the tendon-preparation, on the assumption, therefore, that the levulose permeated the whole tendon, and that its rotation is superadded to the rotation pertaining to the tendon (should this not be the case then the rotation would be somewhat smaller than has been adopted here). The adjustment is somewhat uncertain, the section of the tendon does not get completely dark with any position of the analyzer, and its surface is of course not evenly lucid.

We are now going to show that a bundle of uniaxial crystals, wound along a steep helix, and viewed in the direction of the axis of the helix, must, indeed, display optic rotation. This will be readily understood if we conceive such a preparation as a pile of accumulated disks, so to speak extremely thin transverse sections. In each of these *strongly birefringent* disks the optic axis is *slightly inclined from the vertical* on the plane of section, so that the strong double refraction causes for the observer only small phase-differences; the azimuth of that optic axis alters from one disk to the other and rotates 360° over a distance along the tendon, corresponding with one turn of the helical line.

To simplify our reasoning we imagine this model I substituted by a model II: let the *double refraction be small* in each disk, but the optic axis *be parallel to the plane of section*. The optic axes of the successive disks have successively rotated over a small angle, and we have here an analogue to the well-known experiments of REUSCH with piled up micaplates¹⁾. Just as in REUSCH's experiments we have also here to expect optical rotation. The theory of this phenomenon is furnished by SOHNCKE and MALLARD. A extensive review is to be found in MASCART's Manual²⁾. When the total thickness E of the layers is built up of an infinite number of infinitesimally thin plates, disposed in helical succession with a speed e , the optic rotation R will be:

$$R = \frac{\pi E e}{2 \lambda^2} (n'' - n')^2,$$

in which formula λ represents the wavelength of the light, $n'' - n'$ the double refraction in each of the layers. When applying this formula to the case of the helical lines, and going from the vicarious model II back again to model I, we must bear in mind that in case I we have a stronger double refraction $n_o - n_s$, but that our angle of vision with the optic axis is small, so that that double refraction induces only slight phase-differences.

The ordinary wave, and the extraordinary wave, propagating in an

¹⁾ MULLER—POUILLET, II, 1001.

²⁾ M. E. MASCART, *Traité d'Optique*, II, 324, 1891.

uniaxial crystal at a small angle φ with the optic axis, have according to FRESNEL velocities:

$$V' = V_o = a,$$

$$\text{and } V'' = \sqrt{a^2 \cos^2 \varphi + c^2 \sin^2 \varphi} = a + \frac{c^2 - a^2}{2a} \varphi^2.$$

From this it follows that the difference of the refractive indices

$$n' - n'' = V \left(\frac{1}{V'} - \frac{1}{V''} \right) = n\varphi^2 \frac{c-a}{a} = (n_o - n_e) \varphi^2.$$

If the helix is seen in projection, it will present the aspect of a sine curve $y = a \sin \frac{2\pi x}{E}$, with $\operatorname{tg} \varphi = \frac{dy}{dx} = \frac{2\pi a}{E} = \pi k$, k being the relation of the double amplitude to the period of undulation, which may be measured under the microscope.

For steep helices φ is small, and

$$n' - n'' = \pi^2 k^2 (n_o - n_e).$$

Finally we find for the optic rotation caused by uniaxial crystals, wound in the shape of helical lines:

$$R = 180^\circ \times \frac{\pi^4 k^4 E e}{2 \lambda^2} (n_o - n_e)^2.$$

In our case $E = 3 \text{ mm.}$, $e = 0.04 \text{ mm.}$, $\lambda = 0.00056 \text{ mm.}$

Previous measurements of the double refraction in longitudinal sections have already given for our $n_o - n_e$ values of the order of 0.002. Most difficult of estimation is the magnitude k which varies considerably from the one point of the preparation to the other, and of which an estimation was made in our first communication (Meeting Sept. 25, 1926) that corresponds with $\frac{0.5}{\pi} = 0.16$ in the present notation. When calculating the value to be assigned to k in order to explain the value observed for the rotation R , we find 0.12.

This value is of quite the same order of magnitude as the one previously estimated by us. This is the more satisfactory as the determination of k is attended with some difficulties because of its marked variability. We can say that the amount of rotation has, indeed, been estimated theoretically and confirmed experimentally. That we preferred to calculate a value for k from the observed R , rather than the reverse, is accounted for by the fact that k occurs in our formula in the fourth power, so that every error in the determination of k would tell excessively on the result for R . The high exponent of k explains at the same time why the measured rotation turns out to be larger than the one obtained by calculation; in the preparation different values of k occur, of which the observer chooses an average; for the optic rotation, however, the measure is determined by the average value of k^4 , and that value will be greater.

We have also tried to compare the sense of the optic rotation with the sense in which the helix is wound. It may be estimated by a careful observation of the fibres and by tracing them back by means of the screw of the micrometer. The two directions of the rotation seem to be opposed to each other, which would, indeed, conform with the results of REUSCH's experiments, and with what can easily be explained theoretically.

Besides this fine, well-measurable preparation we have examined others with which the measurement of the optic rotation appeared to be well-nigh impossible. This need not surprise us. With the slightest difference in the values of k (which occurs in most preparations) every part of the tendon will cause a different rotation, and there will be nowhere any total extinction. That the examined preparations, however, really consist of optically strongly rotating parts, becomes evident from the fact *that they are bright between crossed nicols, and that they remain so with any rotation of the preparation or the analyzer.*

It still remains for us to say, that we have tried to find the optic phenomena observed by us in tendons, also in striated muscle-fibres.

a. Longitudinal sections of muscles display magnificent diffraction spectra fairly agreeing in dispersion with those of ROWLAND's gratings.

b. But these spectra do not show polarization.

c. In transverse sections no optic rotation was distinguishable. The field remained dark.

From the facts under *b* and *c*, which fairly agree *inter se*, it might be concluded that fundamentally the way in which the diffraction spectra (*a*) arise in muscles, differs from that in tendons.

*From the Histological Laboratory of Amsterdam
and the Physical Laboratory of Utrecht.*

September 1926.

Physics. — *Rigidity of Super-conductive Metals.* By Prof. W. J. DE HAAS and M. KINOSHITA. (Communication N^o. 187*b* from the Physical Laboratory at Leyden).

(Communicated at the meeting of May 28, 1927).

§ 1. Very little is known of the physical properties of super-conducting metals in their super-conducting states. The object of the following experiments was to investigate whether there is any change, sudden or otherwise, in the material affecting its elastic constants in transition from non-super-conductive to super-conductive state. At the same time, it was thought instructive to see how the constants varied at very low temperatures.

§ 2. *Method.* For the sake of simplicity and ease of experiment, the rigidity — or the torsion modulus — of wires of different metals was measured.

Two metals were chosen namely tin and mercury, both undergoing the change of state into super-conductivity at conveniently attainable temperatures.

When the measurement is made by a static method of twisting a wire by a given torque, any change in the angle so twisted may be caused either by a change in the rigidity of the metal itself or by that in its density affecting the radius, or by a combined effect of both. As the coefficient of thermal expansion of metals at very low temperatures is yet to be found, it was not possible to eliminate this effect from the results obtained. Numerical figures of the rigidity given later are, therefore, without this temperature correction.

As a matter of comparison, the rigidity thus defined was measured within a reasonably wide range of temperatures; in the case of tin, from below the super-conductive state to room temperatures; in the case of mercury, to the temperature of liquid air.

A piece of wire under test was placed vertically at the bottom of a cryostat. It was twisted through a long piece of glass tube and a short length of thin phosphor bronze wire, the latter being well out of and above the cryostat so that it was at a room temperature. Angles of twist of both wires were measured simultaneously by reading through a telescope a scale reflected by two mirrors attached to each end of the wires. With a suitable constructive arrangement, these mirrors were brought close together and so adjusted that two images of the scale could be seen just one above the other.

At a scale distance of 10 metres the accuracy of readings was $1/20$ mm. As the angle of twist of a wire by a given torque is inversely proportional to the fourth power of its radius, if only the volume of the metal changed, a change of the order of 5×10^{-7} cm in the radius of a wire of 0.5 mm. diameter could be detected with a scale deflection of 50 cm. ¹⁾.

Let $\varphi_p, \alpha_p, r_p, l_p, \varphi, \alpha, r$ and l be the rigidity, the angle of twist, the radius and the length of the phosphor bronze wire and the test wire respectively. If D is the torque applied,

$$D = \frac{H}{2} \varphi_p \alpha_p r_p^4 \frac{1}{l_p} = \frac{\pi}{2} \varphi \alpha r^4 \frac{1}{l}.$$

Denote

$$C = \varphi_p r_p^4 \frac{1}{l_p}.$$

C can then be considered as a torsion constant of the phosphor bronze wire, the numerical value of which is found by the oscillation method with a relation

$$C = 2\pi \frac{K}{t^2}$$

where K is the moment of inertia, t the period of oscillation.

In some of the experiments, a spiral spring of phosphor bronze wire was used in place of the straight wire. The torsion constant C of such a spring can also be found in a similar way.

The rigidity φ of the wire is then given by

$$\varphi = C \frac{l}{r^4} \cdot \frac{\alpha_p}{\alpha}.$$

When the temperature correction to r is not considered, the accuracy of the measurement of the variation of φ rests on that of the scale readings corresponding to α_p and α .

By taking a mean of number of readings in twisting the wire to the right and to the left at one temperature for an angle of 100 mm scale reading both sides of the zero point, it was possible to estimate the relative value of φ to five figures.

During the whole series of experiments, great care was taken not to

¹⁾ If n is the scale reading, we have

$$\frac{(r + \delta r)^4}{r^4} = \frac{n}{n + \delta n} \quad \text{or} \quad \delta r = \frac{r \delta n}{4n}$$

For $n = 50$ cm. $\delta n = 1/20$ mm and $r = 1/4$ mm,

$$\delta r = \frac{1}{40} \cdot \frac{1}{200} \cdot \frac{1}{4} \cdot \frac{1}{50} = 6 \cdot 10^{-7} \text{ cm.}$$

exceed the elastic limit of torsion so that the original condition of the wires might be preserved. The amount of twist was kept within $10''$ per diameter length at liquid helium and liquid hydrogen temperatures. Above liquid air temperatures it did not exceed $5''$. There was practically no creep in bringing back the zero point.

Apart from the questions of elastic limit and creep, however, there was a kind of hysteresis effect in raising and lowering the temperature.

When two or more different cooling agents were used to cover a wide range of temperature, it was found necessary, therefore, to vary it only in one direction. When, for instance, the range was required from liquid helium to room temperatures, using liquid hydrogen and liquid air as intermediate media, liquid hydrogen was introduced into the cryostat as soon as liquid helium was evaporated out of it. In this procedure hydrogen snow was generally observed inside the cryostat which showed that the temperature did not rise above that of liquid hydrogen. In similar manner the temperature was raised from liquid hydrogen to liquid air and then to room air temperatures.

Determination of the absolute value of the rigidity was not within the scope of the present experiments. Although its numerical values are given in some of the results, they have little bearing on their true values except that they show the order of magnitude at different temperatures.

§ 3. Results.

1. Tin.

Extruded wires of KAHLBAUM tin of 0.53 mm. diameter were used.

As an arbitrary fixed point on the temperature scale, 0°C. was taken and numerical values of the rigidity measured at other temperatures were divided by those at 0°C. to give a clear view on the variation.

To obtain the value at 0°C. ($\varphi_{0^{\circ}\text{C.}}$), an independent experiment was made by using alcohol as a cooling medium. From the result shown in Table 1, it was found by interpolation

$$\frac{\varphi_t}{\varphi_{0^{\circ}\text{C.}}} = \left(\frac{a_p}{a}\right)_t \bigg/ \left(\frac{a_p}{a}\right)_{0^{\circ}\text{C.}} = 1 - 0.0043 t$$

where φ_t is the value obtained at $t^{\circ}\text{C.}$ This gave $\varphi_{0^{\circ}\text{C.}}$ with a sufficient accuracy between -10°C. and $+20^{\circ}\text{C.}$

The state of super-conductivity is rendered non-super-conductive by an action of a magnetic field. Examinations were repeated several times with different pieces of the wire at the super-conductive temperatures to see if a magnetic field affected the scale readings. The whole cryostat was encased in a long solenoid which produced a reasonably uniform field of 21 Gauss per Ampère in its centre where the tin wire was coaxially placed. Different strengths of the field between 40 and 350 Gauss

TABLE 1. Tin wire n^o. 1. Diameter 0.53 mm. Length 120 mm.

Θ	α_p/x
32.0°C.	0.455
18.0	0.476
7.0	0.500
- 5.0	0.526
-17.0	0.555

which were more than sufficient to destroy the super-conductivity, were tried at all temperatures above and below the threshold value without any effect which shows that the rigidity remained apparently undisturbed in the transition from non-super-conductive to super-conductive state and vice versa.

In view of the threshold value of super-conductive temperature of tin changing under a tensile stress ¹⁾, it was also tried to find if a similar effect is obtained by twisting. For this purpose, a special arrangement was made to measure the electrical resistance of the wire. An electric current of 250 m. Amp. was allowed to pass through the whole system of suspension from the phosphor bronze wire to the tin wire. The resistance was then measured by the potential difference between the terminals of the latter. A potential lead from the upper end of the tin wire was taken through a mercury cup to avoid a torque. At the critical temperatures, at which the resistance was disappearing, the wire was twisted and untwisted, right and left, up to 3.5' per diameter length which was far beyond its elastic limit. The maximum shearing stress calculated on the surface of the wire was 1.3 Kgm per sq. mm. ²⁾ No deviation of the resistance was, however, detected. It has to be taken in mind that the shearing stress is not a homogeneous force as the tensile stress.

The results of the measurements are given in Tables 2, 3 and 4, and Figs. 1, 2 and 3.

Incidentally, it can be remarked that the rigidity varies little, if not increasing, from the temperatures of liquid hydrogen to those at which the metal is super-conductive.

There was a marked difference in the value taken at the intermediate temperatures of liquid air in raising and lowering the temperature as seen in Fig. 2c and Fig. 3c. Similar effect was also noticeable at liquid hydrogen temperatures. At a given temperature, the metal appeared to be softer when the temperature was rising which is just the reverse of the usual hysteresis phenomena.

¹⁾ G. J. SIZOO and H. KAMERLINGH ONNES, Leiden, Comm. 180b, p. 16.

²⁾ The tensile stress used by SIZOO was about 2.5 Kgm. per sq. mm.

There seems to be a change in the rate of variation between 200° Abs. and 250° Abs. This is analogous to YOUNG's modulus of metals in which a similar change occurs at about half the absolute temperature of fusion ¹⁾.

TABLE 2. Tin wire n^o. 2. Diameter 0.53 mm. Length 44.4 mm.

Order of observation	Date	Bath	T	α_p/α	$\varphi/\varphi_{0^\circ\text{C.}}$
1	Nov. 11	Liquid Ethylene	169.5	0.672	1.238
2	"		160	0.677	1.246
3	"		150	0.682	1.256
4	"		141	0.694	1.278
5	"		154.5	0.677	1.246
6	"		170	0.672	1.238
7*)	Nov. 13	Room air	287	0.510	0.940
8	Nov. 25	Liquid methyl chloride	190.5	0.649	1.189
9	"		214	0.625	1.145
10	"		231	0.619	1.134
11	"		237	0.609	1.115
12	"		245	0.597	1.094
13	"		249	0.585	1.071
14	"		237	0.602	1.102
15	"		216.5	0.625	1.145
16**)	Nov. 26	Water	285	0.521	0.955
17	"	Room air	290	0.506	0.927

*) From this observation
 $\varphi_{0^\circ\text{C.}} = (\alpha_p/\alpha)_{0^\circ\text{C.}} = 0.543.$

**) From this observation: $\varphi_{0^\circ\text{C.}} = 0.546.$

2. Single crystal tin.

The wire used was made in the laboratory and measured 121 mm. in length, 0.94 mm. in average diameter. The orientation of the crystal faces was 001, 010 and 100 planes making angles of 94°, 122° and 32° respectively to the axis of the wire.

The result is shown in Table 5 and Fig. 4.

¹⁾ J. P. ANDREWS, Phil. Mag., 50, p. 668, 1925.

TABLE 3. Tin wire n⁰. 3. Diameter 0.533 mm. Length 111.2 mm.
Phosphor bronze spring torsion const. $C = 19800$.

Order of observation	Date	T	α_p/α	$\varphi/\varphi_{0^\circ\text{C.}}$	φ in dyne/cm ²
1	May 21	4.21 K.	0.550	1.368	2.40 · 10 ¹¹
2	"	3.41	0.550	1.368	2.40
3	"	3.79	0.548	1.363	2.40
4	"	3.92	0.549	1.366	2.40
5	"	2.76	0.551	1.371	2.41
6	"	20.4	0.549	1.366	2.40
7	"	14.3	0.551	1.370	2.40
8	"	20.4	0.548	1.363	2.40
9	"	79	0.519	1.291	2.27
10 *)	May 22	287	0.378	0.940	1.65
11	"	82	0.531	1.320	2.32

*) From this observation: $\varphi_{0^\circ\text{C.}} = (\alpha_p/\alpha)_{0^\circ\text{C.}} = 0.402$.

TABLE 4. Tin wire n⁰. 4. Diameter 0.533 mm. Length 85.0 mm.
Phosphor bronze spring torsion const. $C = 4620$.

Order of observation	T	α_p/α	$\varphi/\varphi_{0^\circ\text{C.}}$	φ in dyne/cm ²
1	4.21 K.	3.2268	1.3529	2.51 · 10 ¹¹
2	3.90	3.2273	1.3532	2.51
3	3.70	3.2278	1.3534	2.51
4	3.20	3.2273	1.3532	2.51
5	2.34	3.2278	1.3534	2.51
6	20.4	3.2268	1.3529	2.51
7	14.2	3.2373?	1.3573?	2.52?
8	20.3	3.2213	1.3505	2.51
9	73	3.040	1.2746	2.37
10 *)	288.5	2.220	0.9308	1.73
11	80	3.122	1.3090	2.43
12 *)	290	2.218	0.9300	1.73

*) From these two observations $\varphi_{0^\circ\text{C.}} = (\alpha_p/\alpha)_{0^\circ\text{C.}} = 2.385$.

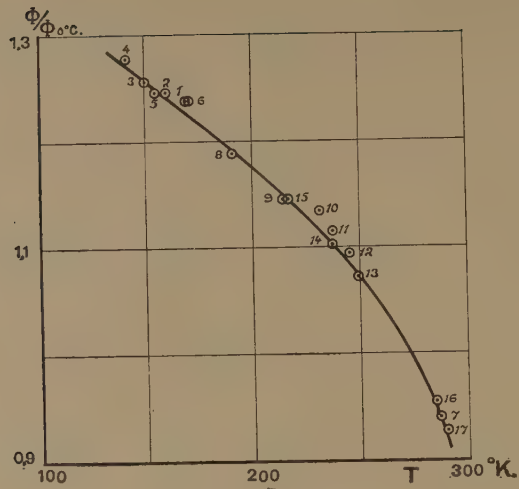


Fig. 1. Tin n°. 2.

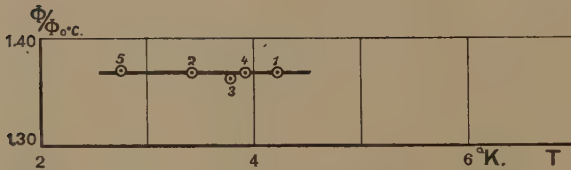


Fig. 2a. Tin n°. 3.

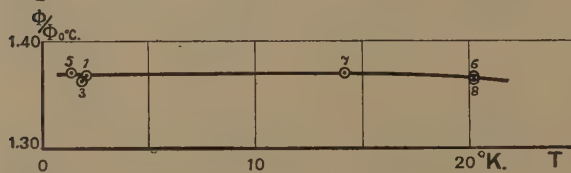


Fig. 2b. Tin n°. 3.

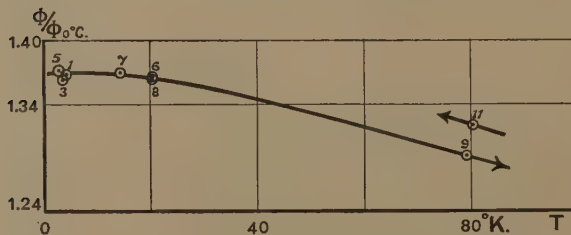


Fig. 2c. Tin n°. 3.
Point 11 from the
chamber temperat.
Point 9 to the
chamber temperat.

Fig. 2.

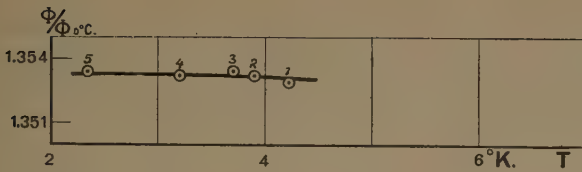


Fig. 3a. Tin n°. 4.

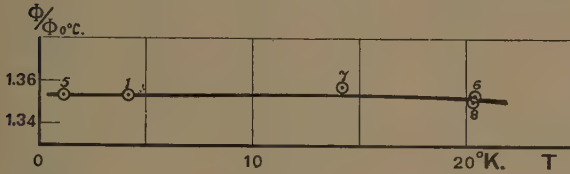


Fig. 3b. Tin n°. 4.

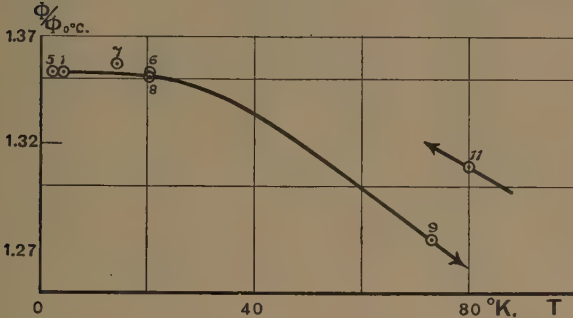
Fig. 3c. Tin n°. 4.
Point 11 from the
chamber temperat.
Point 9 to the
chamber temperat.

Fig. 3.

On account of the cross section of the wire being more or less irregular, only the relative values are given.

The general feature of the variation was very much the same as that of ordinary tin. There was no change in the rigidity in transition from non-super-conductive to super-conductive state.

3. Mercury.

A thin thread of mercury in a paraffin cast was dipped slowly into liquid air. The whole mass of paraffin was then broken away thus leaving a straight piece of mercury wire. Due probably to the process of cooling, a wire so made is apparently of single crystal judging by the way it stretched when pulled. It showed practically no sign of creep at low temperatures and remained very soft even at those of liquid helium.

The result of the experiments, which is given in Tables 6 and 7 and Figs. 5 and 6, shows that the rigidity increases steadily as the temperature drops, even in its super-conductive state¹⁾.

¹⁾ See W. J. DE HAAS, G. J. SIZOO and H. KAMERLINGH ONNES: Leiden Communication 180d, pp. 58, 69 and 75.

At super-conductive temperatures, if rendered non-super-conductive, the electrical resistance of mercury decreases steadily as the temperature is lowered.

TABLE 5. Single crystal tin wire. Diameter (average), 0.94 mm. Length 121 mm.

Order of observation	T	α_p/α	$\varphi/\varphi_{16^\circ\text{C.}}$
1	4.2 K.	5.1324 ?	1.2197 ?
2	3.9	5.1304	1.2194
3	3.7	5.1292	1.2189
4	3.3	5.1298	1.2191
5	2.7	5.1282	1.2187
6	2.2	5.1296	1.2190
7	3.0	5.1300	1.2191
8	4.2	5.1286	1.2188
9	20.4	5.1240	1.2177
10	14.4	5.1268	1.2183
11	20.4	5.1208	1.2169
12	77.0	5.0043	1.1893
13 *)	289.5	4.198	1.00
14 *)	288.5	4.218	
15	83.0	5.0040	1.1892

*) Interpolated from these two observations (at 16°C.)
 $\varphi_{16^\circ\text{C.}} = (\alpha_p/\alpha)_{16^\circ\text{C.}} = 4.208.$

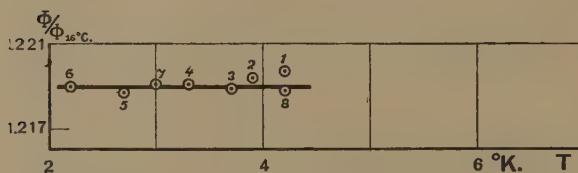


Fig. 4a. A crystal tin wire.

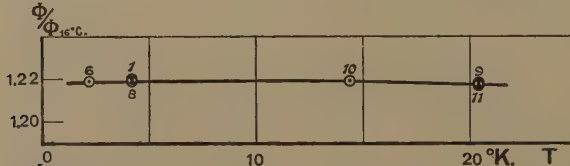


Fig. 4b. A crystal tin wire.

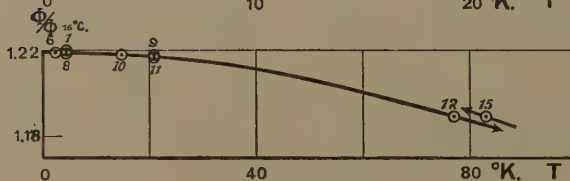


Fig. 4c. A crystal tin wire.

Point 15 from the chamber temperatur.

Point 12 to the chamber temperatur.

Fig. 4.

Again there was no apparent effect of the super-conductive state on the rigidity.

§ 4. *Discussions.* The rigidity of tin and mercury remained apparently undisturbed by the change of state into super-conductivity. It is improbable

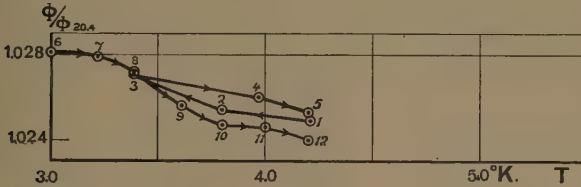


Fig. 5a.
Mercury n⁰. 1.

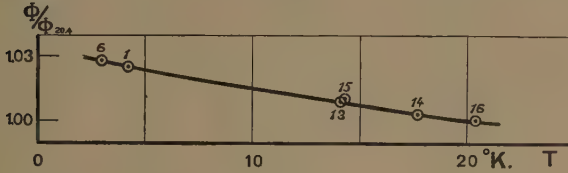


Fig. 5b.
Mercury n⁰. 1.

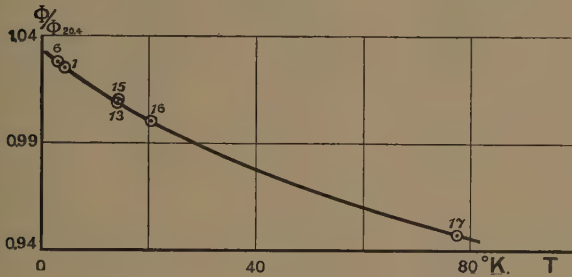


Fig. 5c.
Mercury n⁰. 1.

Fig. 5.

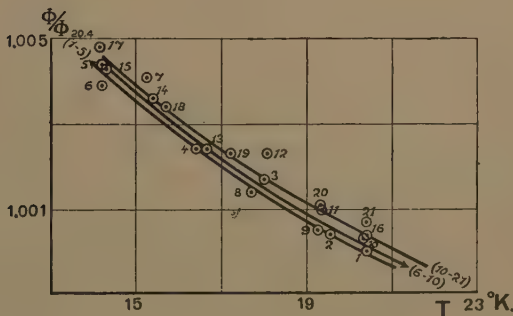


Fig. 6. Mercury n⁰. 2.

that there occurred some such change in the volume of the metals in the transition that it just counterpoised a change in the rigidity thus leaving its apparent value — uncorrected to volume expansion — unchanged

within the accuracy of the measurement. Until more is known about the volume contraction at these low temperatures, no definite statement can be made. At least it can be said that by the arrangement with which a change of the order of 5×10^{-7} cm. in the radius could be detected, no

TABLE 6. Mercury wire n^o. 1. Diameter ca. 0.85 mm. Length 107 mm.

Order of observation	T	α_p/α	$\varphi/\varphi_{20^{\circ}.4 \text{ K.}}$
1	4.21 K.	3.2407	1.0249
2	3.80	3.2422	1.0254
3	3.39	3.2476	1.0271
4	3.97	3.2444	1.0260
5	4.20	3.2421	1.0253
6	3.00	3.2508	1.0281
7	3.22	3.2503	1.0279
8	3.39	3.2480	1.0272
9	3.61	3.2430	1.0256
10	3.80	3.2399	1.0247
11	4.00	3.2398	1.0246
12	4.20	3.2378	1.0240
13	14.1	3.1888	1.0085
14	17.7	3.1716	1.0031
15	14.3	3.1945	1.0103
16	20.4	3.1620	1.0000
17	77.5	2.944	0.9469

positive result was obtained. This accuracy is about the same as that of the method of optical interference.

It is also inconceivable that a change in the rigidity was counteracted by a change of volume caused by the magnetic field which was applied to destroy the state of super-conductivity (magnetostriction).

The effect of a tensile stress is to render a super-conductive metal more favourable to super-conductivity ¹⁾. The resistance begins to disappear at a higher temperature. By twisting a wire, the shearing stress and also the tensile stress are the greatest on the surface. If the mechanism

¹⁾ G. J. SIZOO and H. KAMERLINGH ONNES, Leiden Comm. 180b.

of a super-conductor lies only on its thin surface layer, a twist should be expected to affect the threshold temperature more easily than a pull. Up to the amount of torque applied, by which the elastic limit of torsion was exceeded by far, no change could be noticed.

TABLE 7. Mercury wire n^o. 2. Diameter ca 0.85 mm. Length 107.5 mm.

Order of observation	T	α_p/α	$\varphi/\varphi_{20^{\circ}.4 \text{ K.}}$
1	20.40 K.	2.3920	1.0000
2	19.53	2.3930	1.0004
3	18.00	2.3962	1.0017
4	16.41	2.3978	1.0024
5	14.18	2.4025	1.0044
6	14.22	2.4014	1.0039
7	15.26	2.4019	1.0041
8	17.71	2.3952	1.0014
9	19.25	2.3932	1.0005
10	20.33	2.3927	1.0003
11	19.31	2.3946	1.0010
12	18.07	2.3975	1.0023
13	16.64	2.3978	1.0024
14	15.40	2.4006	1.0036
15	14.30	2.4023	1.0043
16	20.39	2.3930	1.0004
17	14.14	2.4036	1.0048
18	15.69	2.4000	1.0034
19	17.19	2.3975	1.0023
20	19.28	2.3947	1.0011
21	20.39	2.3937	1.0007

Similar results are expected from the other super-conductive metals.

It is likely that other elastic constants will also remain unchanged in transition from non-super-conductive to super-conductive state if measured within the same order of accuracy as obtained in the present experiment.

The authors wish to express their sincere thanks to Mr. J. VOOGD and Dr. H. R. WOLTJER for their valuable assistance.

Chemistry. — *Equilibria in systems, in which phases, separated by a semi-permeable membrane* XXII. By F. A. H. SCHREINEMAKERS.

(Communicated at the meeting of May 28, 1927).

Influence of the temperature on osmotic systems.

In the communications XVIII—XXI we have discussed the influence of a change of pressure on osmotic equilibria and on the osmotic pressure; now we shall discuss briefly the influence of a change of temperature.

In communication XVIII we have a.o. deduced the following. If we have a system $E(P)$ viz. a system (of one or more phases) under the pressure P and we bring this, while the temperature and the total composition remain constant, under the pressure $P + dP$, then arises a new system $E'(P + dP)$ the phases of which can differ a little in composition from those of the first system. If we represent the O.W.A. and the total thermodynamical potential of the first system by ξ and Z , then we have found:

$$\xi = - \frac{\partial Z}{\partial w} \quad . \quad . \quad (1^a) \qquad d\xi = - \Delta V_E \cdot dP \quad . \quad . \quad (1^b)$$

The O.W.A. of the system $E(P)$ is defined by (1^a); the change $d\xi$ which the O.W.A. of this system endures, when it passes into the new system $E'(P + dP)$, is defined by (1^b). We found for the meaning of ΔV_E that $\Delta V_E \cdot \delta w$ represents the change, which gets the total volume of the system E , of this takes in δw quantities of water. If we call ΔV_E the osmotic increase of volume of system E , then follows from (1^b):

with increase of pressure the O.W.A. of a system becomes smaller, when its osmotic increase of volume is positive and greater, when this is negative.

A corresponding property is valid for the change in temperature of a system. If we bring the system $E(T)$ viz. a system of the temperature T , while the pressure and total composition remain constant, to the temperature $T + dT$, then we find:

$$\xi = - \frac{\partial Z}{\partial w} \quad . \quad . \quad (2^a) \qquad d\xi = \Delta H_E \cdot dT \quad . \quad . \quad (2)$$

The change of the O.W.A. which the system endures, if we bring its temperature from T to $T + dT$, is defined by (2^b). For the meaning of ΔH_E we find, that $\Delta H_E \cdot \delta w$ is the change, which the total entropy of the system endures, if it takes in δw quantities of water. We shall call ΔH_E the osmotic increase of entropy of the system E .

When the systems E_1 and E_2 have the same O.W. A. at the temperature T , then exists the osmotic equilibrium:

$$(E_1 | E_2)_T \dots \dots \dots (3)$$

If we bring the temperature to $T + dT$ then these systems change their O.W. A. with:

$$d\xi_1 = \Delta H_{E_1} \cdot dT \dots (4^a) \quad d\xi_2 = \Delta H_{E_2} \cdot dT \dots (4^b)$$

If we take $\Delta H_{E_1} > \Delta H_{E_2}$ then, with increase of temperature, the O.W. A. of the left system becomes greater than that of the right system; consequently water must diffuse from right to left. We shall represent this by:

$$\Delta H_{E_1} > \Delta H_{E_2} \quad [E_1 \leftarrow E_2]_{T+dT} \dots \dots \dots (5)$$

in which dT is positive. Consequently we may say:

with increase of temperature water diffuses towards the system, which has the greatest increase of osmotic entropy;

or also: with increase of temperature water diffuses in such direction that the total entropy of the whole system increases.

We shall represent by $Q_{1.2} \delta w$ the quantity of heat, which is taken up, when δw quantities of water diffuse from a system E_1 towards a system E_2 ; we call $Q_{1.2}$ the heat of diffusion of water from system E_1 towards system E_2 ; the heat of diffusion of water from E_2 towards E_1 is then $Q_{2.1} = -Q_{1.2}$. We now have:

$$\Delta H_{E_1} - \Delta H_{E_2} = \frac{Q_{2.1}}{T} = -\frac{Q_{1.2}}{T} \dots \dots \dots (6)$$

Therefore we can say also: with increase of temperature water diffuses in an osmotic equilibrium in that direction, in which the heat of diffusion is positive or in other words: that heat is absorbed.

We now can deduce the influence of a change of temperature in a corresponding way as in the communications XVIII—XXI, in which is discussed the influence of the pressure on osmotic systems; we shall discuss only some cases.

We take at the temperature T the liquid c of fig. 1 Comm. XVIII; all liquids, which have at this temperature T the same O.W.A. as this liquid c are situated on the isotonic curve acb going through point c . As the liquid q is, therefore, at the temperature T isotonic with the liquid c , we have the osmotic equilibrium:

$$[L_c | L_q]_T \quad \text{fig. 1. XVIII} \dots \dots \dots (7)$$

If we represent the osmotic increases of entropy by ΔH_c and ΔH_q , then, if we bring the temperature to $T + dT$, the O.W.A. at the left and at the right of the membrane, changes with:

$$d\xi_c = \Delta H_c \cdot dT \dots (8^a) \quad d\xi_q = \Delta H_q \cdot dT \dots (8^b)$$

so that in general the two liquids are no more in osmotic equilibrium with one another. If we put $\Delta H_c > \Delta H_q$ then consequently water diffuses from q towards liquid c . We can represent this by:

$$\Delta H_c > \Delta H_q \quad \left[L_c \leftarrow L_q \right]_{T+dT} \quad \text{fig. 1. XVIII} \quad . \quad . \quad (9)$$

in which dT is positive.

As in the osmotic system (9) the O.W.A. of the liquid c is greater than that of the liquid q , we can get an osmotic equilibrium of the temperature $T + dT$ by replacing the liquid q by a liquid q' with an O.W.A. which is somewhat greater, so that the O.W.A. on both sides of the membrane becomes equal again. Amongst others we can get such a liquid q' , as we have seen in previous communications, by withdrawing a little water from the liquid q . We then have the osmotic equilibrium:

$$\left[L_c \mid L_{q'} \right]_{T+dT} \quad \text{fig. 1. XVIII} \quad . \quad . \quad . \quad (10)$$

in which the liquid q' can be represented by the point q' of fig. 1 XVIII, situated on the line Wq .

Also the following appears a.o. from this. If the liquids, which are isotonic with the liquid c at the temperature T , are represented by curve acb , then the liquids, which are isotonic with liquid c at the temperature $T + dT$, will be represented by a curve $a_1 c b_1$ (fig. 1 XVIII) which does not coincide with acb .

We are able to deduce the above yet also in another way. For the osmotic equilibrium (7) is valid the relation:

$$\left(\zeta - x \frac{\partial \zeta}{\partial x} - y \frac{\partial \zeta}{\partial y} \right)_c = \left(\zeta - x \frac{\partial \zeta}{\partial x} - y \frac{\partial \zeta}{\partial y} \right)_q \quad . \quad . \quad . \quad (11)$$

For the osmotic equilibrium (10), of which we assume that the liquid q' differs infinitely little (viz. dx and dy) from the liquid q , then is valid, as follows from (11).

$$\left(\eta - x \frac{\partial \eta}{\partial x} - y \frac{\partial \eta}{\partial y} \right)_c dT = \left(\eta - x \frac{\partial \eta}{\partial x} - y \frac{\partial \eta}{\partial y} \right)_q dT + (rx + sy) dx + \left. \begin{aligned} & + (sx + ty) dy \end{aligned} \right\} \quad (12)$$

in which, as we deduce easily, the coefficients of dT are the osmotic increases of entropy of the liquids c and q . As we can satisfy (12) for a definite value of dT still by an infinite number of values of dx and dy , there exists, therefore, in the vicinity of liquid q (fig. 1 XVIII) an infinite number of liquids q' , which are all isotonic with the liquid c at the temperature $T + dT$. Of all those liquids we take a liquid q' , which can arise from q by taking in or giving a little water, so that it is situated in fig. 1 XVIII anywhere on the line Wq ; then dx and dy must satisfy:

$$dx : x = dy : y = d\lambda \quad . \quad . \quad . \quad (13)$$

We now can write for (12):

$$(\Delta H_c - \Delta H_q) dT = (rx^2 + 2sxy + ty^2) d\lambda. \quad (14)$$

by which the value of $d\lambda$ is defined. If we assume, in accordance with above, that ΔH_c is greater than ΔH_q and if we take dT positive, then it follows from (14) that $d\lambda$ is positive also. As it follows from (13) that dx and dy are positive also now, it follows that this liquid q' must be situated, as is drawn in fig. 1 XVIII, viz. further from the point W than the point q .

The above is valid no more when point c and consequently also point q are situated in the vicinity of point W , so that the isotonic curves are straight lines, which cut off equal parts of the sides WX and WY ; the isotonic curves of the temperatures T and $T + dT$ going through point c , will coincide then. This follows a.o. also from (12); for infinitely small values of x and y the coefficients of dT in (12) approach both the entropy of the pure water, so that their difference approaches zero. As s remains finite, sx and sy approach zero; rx and ty , however, approach both RT . Then follows from (12):

$$0 = dx + dy \quad (15)$$

from which the above follows at once. We are able to find this also by substituting in (11):

$$\zeta = \varphi + RT(\log x + \log y) \quad (16)$$

φ and its derivations with respect to x and y then remain finite for infinitely small values of x and y . We then find:

$$\left[\varphi - x \frac{\partial \varphi}{\partial x} - y \frac{\partial \varphi}{\partial y} - RT(x + y) \right]_c = \left[\varphi - x \frac{\partial \varphi}{\partial x} - y \frac{\partial \varphi}{\partial y} - RT(x + y) \right]_q$$

For infinitely small values of x and y follows again from this the equation (15).

In order that two systems E_1 and E_2 can be in osmotic equilibrium with one another, it is not sufficient that they have the same $O.W.A.$, but also their temperature must be the same; the pressure of the two systems, however, can be as well equal as different. Consequently, provided that the temperature of the separate systems is the same, an osmotic equilibrium:

$$(E_1)_{P_1} \mid (E_2)_{P_2} \quad (17)$$

may exist, in which P_1 and P_2 can represent the same pressures or not. Let us take f.i. the osmotic equilibrium:

$$L_P \mid L_P \quad (18)$$

in which on both sides of the membrane the same liquid and the same pressure. If we bring the pressure on the right side of the membrane to $P + dP$, then we can compare the $O.W.A.$ on both sides of the membrane

with one another; at the right side it increases with respect to the left with $-\Delta V_L \cdot dP$; if we take dP positive, then the *O.W.A.* at the right side of the membrane decreases, therefore; consequently in the osmotic system:

$$L_P \leftarrow L_{P+dP} \dots \dots \dots (19)$$

a little water diffuses from the right towards the left. By changing a little the composition of one or of both liquids, so that the *O.W.A.* becomes equal again on both sides of the membrane, we are able to prevent the diffusion of water; we then get, instead of (19) one of the osmotic equilibria:

$$L_P^{(1)} \mid L_{P+dP} \quad L_P \mid L_{P+dP}^{(2)} \quad L_P^{(3)} \mid L_{P+dP}^{(4)} \dots \dots \dots (20)$$

in which the liquids $L^{(1)}$ $L^{(2)}$ $L^{(3)}$ and $L^{(4)}$ differ a little in composition with the liquid L .

If we take, however, the osmotic equilibrium:

$$(E_1)_T \mid (E_2)_T \dots \dots \dots (21)$$

in which the pressure on both sides of the membrane is equal or not and if we bring the temperature on the right side of the membrane to $T + dT$ then we get:

$$(E_1)_T \mid (E_2)_{T+dT} \dots \dots \dots (22)$$

which is not an osmotic system, however, and which never can pass into an osmotic equilibrium, in whatever way we change the composition and the pressure of the separate systems. Consequently we cannot compare the *O.W.A.* of both the separate systems with one another. Of course this is really the case, when we bring the temperature on both sides of the membrane to $T + dT$, so that we get the osmotic system:

$$(E_1)_{T+dT} \mid (E_2)_{T+dT} \dots \dots \dots (23)$$

While at change of pressure the osmotic changes of volume ΔV_{E_1} and ΔV_{E_2} may assume a part separately, this is not the case at a change of temperature with the osmotic changes of entropy ΔH_{E_1} and ΔH_{E_2} ; as viz. the temperature must be always the same on both sides of the membrane, and must change therefore, also always with a same amount, here always occurs the difference $\Delta H_{E_1} - \Delta H_{E_2}$ and also, therefore, the diffusion heat $Q_{2,1}$.

In communication XIX we have deduced the osmotic change of volume ΔV_E for different systems E ; in a corresponding way we are able to find the osmotic change of entropy ΔH_E . If we take f.i. the system:

$$E = (Y + L_w)_T \quad \text{fig. 1. III} \dots \dots \dots (24)$$

then we find for the osmotic change in entropy:

$$\Delta H_E = \frac{\eta - y \eta_y}{1 - y} \dots \dots \dots (25)$$

in which η represents the entropy of the liquid and η_y that of the solid substance Y . If we take one of the systems:

$$E_1 = (Y + L_a)_T \quad \text{fig. 1. III} \quad E_2 = (X + Y + L_c)_T \quad \text{fig. 1. V}$$

$$E_3 = (Y + H + L_d)_T \quad \text{fig. 2. V}$$

then the osmotic changes in entropy are defined by:

$$\Delta H_{E_1} = \eta + \frac{\partial \eta}{\partial x} + \frac{s}{t} \left(\eta_y - \eta - \frac{\partial \eta}{\partial y} \right)$$

$$\Delta H_{E_2} = \frac{\eta - x \eta_x - y \eta_y}{1 - x - y} \quad \Delta H_{E_3} = \frac{\eta_H - \beta \eta_y}{1 - \beta}$$

in which H represents a hydrate with the composition $\beta \text{ Mol. } Y + (1 - \beta) \text{ Mol. } W$; the entropies of the solid substances X, Y and H are indicated by η_x, η_y and η_H .

We assume that fig. 1. III is valid for a definite temperature T and pressure P ; the *O.W.A.* of the solid substance Y at this T and P then is the same as that of the liquid w saturated with solid Y and also therefore, equal to the *O.W.A.* of every liquid of the isotonic curve wm . If we take on this curve an arbitrary liquid e , then we have, therefore, the osmotic equilibrium:

$$(Y + L_w)_T \mid (L_e)_T \quad \text{fig. 1. III} \quad (26)$$

with which the index P is omitted, as further we shall keep the pressure constant. We now bring the pressure on both sides of the membrane to $T + dT$, while we keep constant the composition of the both separate systems; we then get the osmotic system:

$$(Y + L_w)_{T+dT} \mid (L_e)_{T+dT} \quad (27)$$

Although the total composition at the left side of the membrane remains constant, yet the liquid w will change a little its composition by dissolving or depositing solid Y ; we can imagine this new liquid w' to be represented in fig. 1. III by a point w' (on the side WY) in the vicinity of point w .

If we represent the *O.W.A.* and the osmotic increase of entropy of the left system by ξ_E and ΔH_E and those of the right system by ξ_e and ΔH_e , then the *O.W.A.* on the left and the right side of the membrane increases with:

$$d\xi_E = \Delta H_E dT \quad d\xi_e = \Delta H_e \cdot dT \quad (28)$$

in general, therefore, (27) forms no more an osmotic equilibrium and a little water must diffuse through the membrane. The direction of this diffusion is defined by the sign of:

$$d\xi_E - d\xi_e = (\Delta H_E - \Delta H_e) dT = \frac{Q_{e,E}}{T} \cdot dT \quad . . . (29)$$

in which $Q_{e,E}$ represents the diffusion-heat of water from the liquid e towards the system $E = Y + L_w$.

We shall divide, into two parts the reaction, which occurs when δw quantities of water are taken in by the system $Y + L_w$; at first viz. we shall mix this water with the liquid L_w and afterwards we shall dissolve $y \delta w$: $(1-Y)$ quantities of solid Y in this liquid. If we represent the mixture-heat of water with the liquids L_w and L_e by Q_w and Q_e and the dissolving-heat of the solid substance Y in the liquid L_w by Q_y , then we have:

$$Q_{e.E} = Q_w - Q_e + \frac{y}{1-y} \cdot Q_y \dots \dots \dots (30)$$

Hence it appears that the diffusion-heat can be as well positive as negative; if the liquid w , however, contains still little water only, so that y approaches the unity, then this diffusion-heat is in general positive. If the liquid w viz. contains still little water only, then the dissolving heat Q_y approaches the melting-heat of the solid substance Y and is positive, therefore; as $1-y$ then is small also, the last term of (30) shall get such a great positive value that the total value of (30) is positive also.

We now take $Q_{e.E}$ positive. Then follows from (29) that in the osmotic system (27) the *O.W.A.* at the left side of the membrane is greater than that at the right side; consequently a little water must diffuse from right to left. The osmotic system (27) passes by this into the osmotic equilibrium:

$$(Y + L_w)_{T+dT} | (L_e)_{T+dT} \dots \dots \dots (31)$$

The liquid w' at the left of the membrane keeps with this (by dissolving or depositing a little Y) its composition. On account of its giving of a little water the right liquid e passes into a liquid e' , which we can imagine to be represented in fig. 1. III by a point e' on the line We and somewhat further from W than point e .

We are able to deduce the previous in another way also. For the osmotic equilibrium (26) viz. is valid the equation:

$$\left(\zeta - y \frac{\partial \zeta}{\partial y} \right)_w = \left(\zeta - x \frac{\partial \zeta}{\partial x} - y \frac{\partial \zeta}{\partial y} \right)_e \dots \dots \dots (32)$$

which expresses that the *O.W.A.* is equal on both sides of the membrane. As however the liquid w on the left side of the membrane is still also in equilibrium with solid Y we have:

$$\left[\zeta + (1-y) \frac{\partial \zeta}{\partial y} \right]_w = \zeta_y \dots \dots \dots (33)$$

If we represent the difference in composition of the liquids e' and e by dx and dy and that of the liquids w' and w by dy' then the osmotic equilibrium (31) is defined by:

$$\left(\eta - y \frac{\partial \eta}{\partial y} \right)_w dT + (y t)_w \cdot dy' = \left(\eta - x \frac{\partial \eta}{\partial x} - y \frac{\partial \eta}{\partial y} \right) dT + \left\{ \begin{aligned} &+ (rx + sy)_e \cdot dx + (sx + ty)_e \cdot dy \end{aligned} \right\} \dots \dots \dots (32^a)$$

$$\left[\eta + (1-y) \frac{\partial \eta}{\partial y} \right] dT - [(1-y) t]_w \cdot dy' = \eta_y \cdot dT \dots \dots \dots (33^a)$$

of which (32^a) follows from (32) and (33^a) from (33). By elimination on dy' follows:

$$A \cdot dT = (rx + sy)_e dx + (sx + ty) dy \quad . \quad . \quad . \quad (34)$$

in which:

$$A = \left(\frac{\eta - y \eta_y}{1 - y} \right)_w - \left(\eta - x \frac{\partial \eta}{\partial x} - y \frac{\partial \eta}{\partial y} \right)_e \quad . \quad . \quad . \quad (35)$$

It appears from (24) and (25) that the term with the index w in (35) represents the osmotic increase of entropy of the system $Y + L_w$; the term with the index e represents the osmotic increase of entropy of the liquid L_e . Consequently we have:

$$A = \Delta H_E - \Delta H_e = \frac{Q_{e,E}}{T} \quad . \quad . \quad . \quad (36)$$

If we substitute this value of A in (34) and if we use (13), then (34) passes into:

$$\frac{Q_{e,E}}{T} \cdot dT = (rx^2 + 2sxy + ty^2) d\lambda \quad . \quad . \quad . \quad (37)$$

If we take, as above, $Q_{e,E}$ positive, then follows that $d\lambda$ is positive also; hence follows for the position of liquid L_e of (31) the same as we have deduced already above.

In fig. 1 are represented by curve $w_1 v_1$ the solutions saturated with the solid substance Y at the temperature T_1 ; the isotonic curve of the same temperature going through point w_1 is indicated by $w_1 m_1$. For

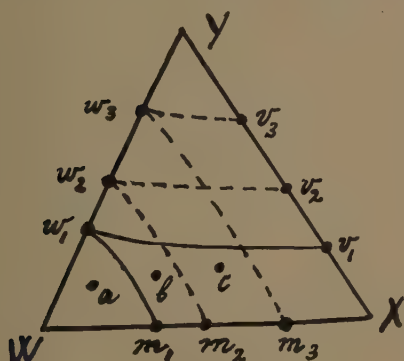


Fig. 1.

fixing the ideas we shall assume now that the solubility of the solid substance Y increases with increase of temperature; if we put $T_1 < T_2 < T_3$ then $w_2 v_2$ and $w_3 v_3$ can represent the saturation-curves of the temperatures T_2 and T_3 ; further we assume that $w_2 m_2$ and $w_3 m_3$ represent the isotonic curves of those temperatures.

The O.W.A. of the solid substance Y is equal to that of the liquid w_1 (at the temperature T_1) and, therefore, also equal to that of every liquid of the curve

$w_1 m_1$. Consequently the solid substance Y has at this temperature a greater O.W.A. than the liquid a , but a smaller O.W.A. than the liquids b and c . In the first of the three systems:

$$[L_a \rightarrow Y]_{T_1} \quad [L_b \mid Y]_{T_1} \quad [L_c \mid Y]_{T_1} \quad \text{fig. 1.} \quad . \quad . \quad (38)$$

water diffuses, therefore, in the direction of the arrow; if there is sufficient solid Y then is formed the osmotic equilibrium:

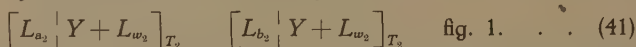
$$[L_{a_1} \mid Y + L_{w_1}]_{T_1} \quad \text{fig. 1.} \quad . \quad . \quad . \quad (39)$$

in which a_1 is the point of intersection of the line Wa with the isotonic curve w_1m_1 . The second and the third system are osmotic complexes, in which nothing happens.

We now bring the three systems (38) to the temperature T_2 ; water will diffuse in the two first systems from left to right; consequently we have:

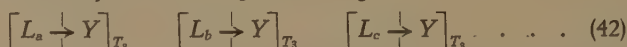


In the third of those systems nothing happens. If a sufficient quantity of solid Y is present, then both the first pass into:



in which a_2 and b_2 are the points of intersection of the lines Wa and Wb with the curve w_2m_2 .

If we raise the temperature to T_3 then we get:



viz. a diffusion of water in each of the three systems. If there is a sufficient quantity of solid Y then arises on the right side of the membrane $Y + L_{w_3}$.

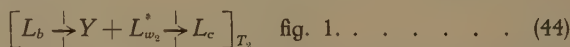
It is apparent from those examples that the diffusing or not of water through the membrane towards a solid substance, also is dependent on the temperature.

We now take the osmotic system:



with a double-membrane. (Compare a.o. also Communication X). Although the O.W.A. of both liquids is different, herein nothing happens, because the O.W.A. of the solid substance Y is smaller than that of each of the liquids. Consequently the double-membrane is impermeable for water.

If we raise, however, the temperature to T_2 then the O.W.A. of the solid substance Y becomes greater than that of the liquid b , while it is smaller than that of the liquid c . At the beginning of the diffusion we then have the system:



If we bring the temperature to T_3 then the O.W.A. of the solid substance Y is greater than that of each of the liquids; at the beginning of the diffusion we then have the system:



As we have discussed already in Communication X it depends on the ratio of the quantities of the phases, etc., which osmotic equilibria will be formed at last from (44) and (45).

It appears from those examples that, therefore, it depends also on the temperature whether a double-membrane is permeable or not.

(To be continued).

Leiden, Laboratory for Inorg. Chem.

Microbiology. — *On Digestion in Protozoa.* By S. L. SCHOUTEN Sc. D.
(Communicated by Prof. C. EIJKMAN.)

(Communicated at the meeting of May 28, 1927)

Our knowledge of the physiology of digestion in Protozoa is comparatively limited. RUD. OEHLER, one of the best investigators during late years, even asserts summarily¹⁾ that there is no physiology of the unicellular organisms. He ascribes this to the lack of pure cultures, so that we do not know e.g. what is digested and what is not; nor what bacteria, when ingested, exert a deleterious effect through their metabolic products. Yet, for our knowledge of the self-purification of water, for instance, a close study of these problems is highly necessary.

Up to now researchers have tried to obtain pure cultures with one accompanying strain of bacteria by washing out repeatedly, centrifugalizing and planting out on solid culture media; by starting from cysts that are purified from adhering bacteria with water or soda-solution; by means of an electric current, whereby the protozoa are allured to the cathode; by inoculation in the centre of an agar-plate, previously planted entirely with a pure culture of a bacterium, in order to get finally, after a repetition of this inoculation, the protozoa that are moving rapidly towards the periphery, in company with that one bacterium.

But the results of all these endeavours are not brilliant. OEHLER, who has particularly applied himself to a solution of this problem, arrived at the conclusion: „Gewiss müssen andere, bessere Verfahren ausgearbeitet werden um Sterilzuchten zu erreichen“²⁾).

I thought fit to employ my apparatus for individual treatment of micro-organisms³⁾.

A drop of the material, from which one wishes to isolate, is placed on the coverslip, as well as a number of drops of one sterile culture medium or other. A confluence is now cautiously effected between the “material drop” and the sterile drop immediately contiguous to it, so that protozoa can swim across to the new, limpid portion of the medium.

With a glass needle at one end bent into a loop (Fig. 1; a rather large loop, in which the protozoa can swim about unscathed) and at low-power magnification they are now fished off from this limpid portion, and transferred to the second drop, then to the third etc. In each of these drops they

¹⁾ Arch. f. Protistenkunde, 1924, p. 288.

²⁾ Ibid. 1919 p. 19.

³⁾ See Kon. Akad. van Wetensch. Verslag van de Wis- en Natuurk. Afd. 24 Dec. 1910.

keep swimming about for some time to be deprived of the accompanying bacteria. Experience will show how many drops they have to pass through. Ultimately they are allowed to multiply in the last drop, after having added at will dead bacteria or living ones in pure culture (initially deposited on the cover-glass). Generally after one or two days it will be possible to transfer a subculture from that drop to a test-tube.

After having thus obtained several pure cultures, I selected one of *Colpidium campylum* to experiment on the rate of digestion. I must postpone a more detailed systematic enquiry until later; we can now only make a dip into the subject here and there, which, however admits of forming an idea about the usefulness of the method.

The protozoa were placed on a coverslip, together with a number of washing-drops, as described above, and also a few drops of ordinary nutrient-gelatin-agar¹⁾ (a mixture of equal parts of nutrient-gelatin and nutrient-agar). Likewise a small lump of pure culture of bacilli whose digestibility I wished to establish, was deposited among the drops.

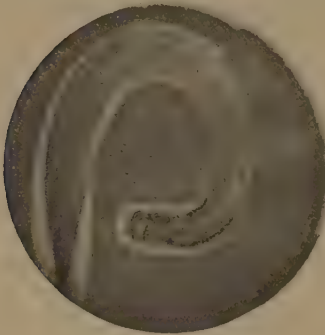


Fig. 1. ($\times 240$)

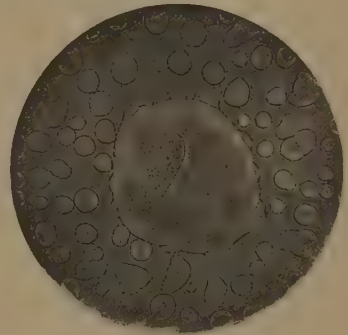


Fig. 2. ($\times 240$)



Fig. 3. ($\times 240$)

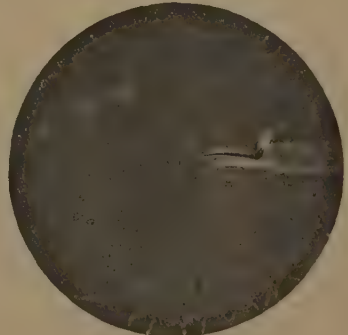


Fig. 4. ($\times 240$)

¹⁾ PRALL, Beitrag zur Kenntniss der Nährböden. Arb. a. d. Kaiserl. Gesundh. 1902, p. 436

After the protozoa had gone through different drops, I finally left them for some hours in a drop, without food. If I found by means of high-power magnification that they did not contain filled gastric vacuoles any more (with a view to this they were just for a short period removed from the large drop and examined in a little one: see fig. 2), they were fed again. At different intervals the protozoa were opened with the dissecting needle (fig. 3), so that the filled vacuoles were isolated (fig. 4)¹⁾.

These were crushed, and after being washed out the bacteria were deposited at the periphery of the gelatin-, or agar-gelatin drops. Whether they were dead, or alive could be seen from the multiplication.

From these experiments it first of all became evident that when *Colpidium* meets only few bacteria, it ignores them, but it devours them with great zest when it comes across a large mass, by preference a big lump, into which it bores its way. Its rapidly rotating movements give one the impression that it is revelling in food.

In a few minutes three or more vacuoles, filled with bacteria, are distinguishable.

In a special examination of the vacuoles it was not difficult to distribute the bacteria over various drops, and to count them. So I counted 127 spores of *B. subtilis* and in a small gastric vacuole 64 typhus bacilli; so at the same time 500 bacteria could easily be devoured.

Opinions differ about the wall of the gastric vacuoles. It is mostly conceived as a differentiation of the entoplasm, originated from a contact of plasm and water, that is ingested together with the food; all this occurs under the influence of surface-tension and colloid-chemical factors²⁾. By means of the pointed needle it could easily be found out that that wall is rather tough; when on the one side I had made a puncture, a slight push at the opposite side would keep the wall intact, but made the bacteria slip through the opening, the one after the other.

Some idea about the rate of digestion was afforded by the following experiments. Our *Colpidium* was very fond of a non-determined subtilis-like bacterium. The feeding itself lasted five minutes. When 10 minutes afterwards the bacteria were deposited in the way described above, they did not grow. When this was done 5 minutes after the feeding, they appeared to be alive yet. As could be anticipated, spores of these bacteria held out much longer in the vacuoles. When I allowed *Colpidium* to live still 30 minutes after the feeding, then excised the vacuoles and left them alone for 15 minutes, and subsequently inoculated the spores, they all of them appeared still to possess germinative power. I even experienced that germination was possible when these periods of 30 and 15 minutes were lengthened resp. to 90 and 60 minutes. Under certain conditions completely

¹⁾ Fig. 4 is somewhat indistinct, especially as regards the gastric vacuoles, owing to the quality of paper on which it is printed.

²⁾ See also: BOZLER, Arch. f. Protistenkunde, 1924, p. 163.

filled vacuoles are extruded ; when next day I examined their spores, they appeared to be still viable.

I also experimented with Anthrax, Coli, and Typhus.

As regards Anthrax let it be mentioned only that the bacteria were not devoured, apparently because they are too large, whereas on the other hand the spores were ingested.

Coli, and Typhus appeared to be much more resistant than the subtilis-like bacterium. I once left *B. coli communis* for 40 minutes after the feeding in the living *Colpidia* ; after this the animals were killed off, so that the gastric vacuoles were again at disposal. Untoward circumstances prevented me from finishing the experiment the same evening ; so the preparation was placed in the refrigerator. After 19 hours the wall of the vacuole was broken and the bacteria appeared to have survived. Typhus bacilli were not killed off after a sojourn of 40 minutes in the living *Colpidia*.

I may be allowed to add a few purely technical remarks :

For successful isolation experiments the surface of the coverslip is of prime importance. I intimated already in my first publication (1899), that the coverslip should be made fatty, so that the drops do not coalesce. It is not immaterial, though, what sort of fat is used for this purpose. In the beginning I used vaselin, though it is no fat at all ; afterwards I took suet, which satisfied me much better. Nonetheless also with this sort of fat I was more or less troubled by coalescence of the drops, especially with high summer temperatures and when a preparation had to serve a rather long time. At last I found out that we wanted suet, still covered by the peritoneum. Probably on its outside there are only the solid fats, while the liquid fats mixed with the solid ones form the interior. The underside of the coverslip is rubbed over the peritoneum, then nearly all the fat is removed by rubbing with a piece of soft linen, and the coverslip is flamed.

Another experience I often made with cultures in moist chambers was that several micro-organisms are rather sensitive to a rise of vapourpressure, that necessarily must originate in these moist chambers in the incubator from their atmosphere being saturated with water-vapour. This has been pointed out before by other investigators ; CLARK ¹⁾, therefore, suggests not to cover the chamber entirely with the coverglass, when it is placed in the incubator, but to leave a slit open, which is closed afterwards. To this end, however, the moist chamber will perhaps have to be taken out of the incubator, so that it will cool down rapidly ; after being closed its temperature will increase again. Moreover through the slit evaporation of the drop can take place. Fig. 5 shows a cross-section of a very simple isobarometric moist chamber.

A square frame of nickle-plated copper with interior dimensions of 14×14 mm., the breadth and the height of the border measuring resp. 4 and 3.5 mm., has been stuck on an objectglass with ordinary sealing-wax

¹⁾ CLARK, On the toxic effect etc. Bot. Gazette, 1899, p. 289.

(fig. 5 to the left, from above). The right sidewall has been filed down obliquely over a distance of 6 mm. on either side, so that its lower part is narrowed. In the middle of this narrowed part a fine slit has been filed, so that the interior of the moist chamber is connected with the open air.

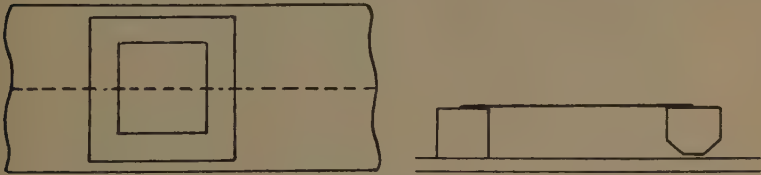


Fig. 5.

Cross-section along the dotted line is seen at the right of the figure, magnified $2\times$. Before closing the chamber with the coverglass a drop of thick oil ($2\frac{1}{2}$ gr. of vaselin to 10 gr. of paraffine liquidum, well mixed on a warm water bath) is placed in the obliquely filed part on the inside. On a rise of the vapour-pressure air-bubbles are seen escaping at the outside of the slit, when the pressure is lessened on the inside. The oil keeps the slit closed.

Utrecht.

*From the Hygienic Institute.
(Director Prof. C. EIJKMAN.)*

Physiology. — *On the Origin of the Radiation substances, and their Action on the Heart.* By Prof. H. ZWAARDEMAKER M. D.

(Communicated at the meeting of May 28, 1927).

In an earlier publication ¹⁾ we reported that in an isolated heart, that has discontinued its pulsation through removal of potassium, radium rays of weak penetrating power, or polonium radiation can generate substances, which, when brought into a small circulation, are able to restore the function of another heart, that has been prepared in the same way.

These substances may appropriately be called *automatins*; if we bear in mind that in the human body alpha-radiation is lacking, and consequently alpha-automatin does not normally occur, the natural automatin will be beta-automatin. It arises from a mother-substance, that is best designated by the name of "automatinogen". It is present throughout the organ that is activated by radiation.

Now it is just the activated substance that can set a heart beating spontaneously for a considerable time, after it has ceased to beat through a paucity of automatin and potassium.

The presence of a mother-substance was suggested to me by the fact that a fluid, that has been perfusing a non-pulsating heart for hours, acquires, after being radiated in a beaker-glass, the same properties as the fluid from a radiated heart.

It may be assumed, that the same occurs in a natural condition i.e. that throughout the organ mother-substance occurs that is activated by the radiation of the potassium contained in the muscle (2—4 β -rays per sec. and per gramme of substance), and that proceeds to the nodal tissue partly through direct diffusion, partly via the circulation (of the blood), where the automatin is eagerly absorbed and sustains the normal automatism.

This state of things, viz. that the mother-substance is contained in the heart-muscle, but that for automatism the action of activated mother-substance is required, is rendered plausible by the following experiments, which were made again in collaboration with Dr. PH. ARONS.

A frog's heart is attached to a Kronecker-canula and the ligature is laid slightly higher than the middle of the ventricle, all that is above the ligature being cut away, so that we have to do only with the apex of the heart without the atrioventricular node. Now, when we radiate this apex, in circulation, with radium, the heart will not pulsate, but the fluid can make a non-pulsating eel's heart resume its beats again.

¹⁾ H. ZWAARDEMAKER, These Proceedings, 30 p. 184 and 420.

If we do not radiate the apex of the heart, but for the rest arrange the experiment in the same way, then the liquid will be inactive.

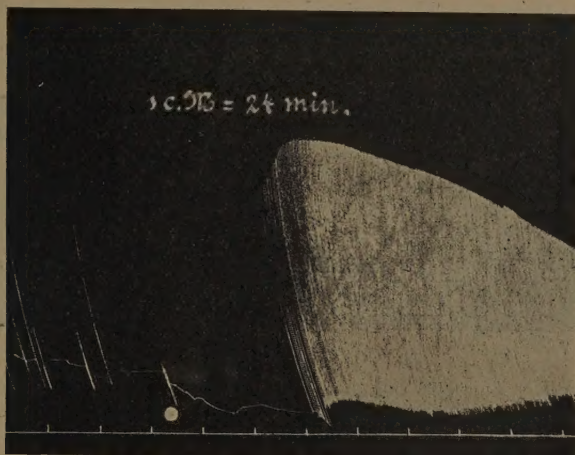


Fig. 1.

Recovery of an eel's heart in circulation, due to a circulating fluid that has been derived from a non-pulsating, but radiated portion of a frog's ventricle (lower two thirds). Commencement of the action at the white speck. One scale-mark of the time-line = 24 min.

So, with adequate radiation, molecules of the mother-substances as well as activated molecules, whose effect on a second heart is illustrated in Fig. 1., pass from the lower two thirds of a motionless frog's heart into the perfusing fluid.

Now that it appears that the mother-substance of the normal automatin is present throughout the heart-muscle, the question arises, whether it also occurs in the skeletal muscle.

This question is set at rest by the following experiment :

Three frogs are deprived of their blood as much as possible through perfusion for three hours with potassium free fluid. Then the muscles of the hindlegs are cut loose from their environment ; the nerves, the vessels and the tendons are removed as much as possible. The muscles are minced finely and shaken in alcohol for half an hour ; after they have been pressed out in a tissue-press, as much as possible, the fluid acquired is filtered. We allow the alcohol to evaporate at a moderate temperature, and wash the residue with ether. Then all is dissolved in potassium-free fluid. If we shake this solution with talc, the talc will absorb mother-substance, as well as the activated substance occasionally present. From the talc we can extract the absorbed substances with alcohol. Again the alcohol is allowed to evaporate and we dissolve the substance in potassium-free fluid.

Half of the extract contained in the beaker-glass is radiated in a thin layer, the other half is not radiated. Both halves are tested separately on a heart that has been brought to a standstill through deprivation of potassium and automatin. The radiated, as well as the non-radiated muscle-extract appears to be active, but the former is always the most powerful.



Fig. 2.

Recovery of an eel's heart in circulation owing to radiated muscle-extract (alcohol, absorption, alcohol, Ringer-solution). Commencement of the action at the white speck. One scale-mark of the time-line = 16 minutes.

This figure instances such a recovery produced by a radiated muscle-extract. The pulsation persisted regularly without flagging for 20 hours, while the heart perfused with the non-radiated half pulsated only 8 hours. Thus, through extraction with alcohol we can obtain from the skeletal muscles of the frog, previously deprived of potassium and automatin, both mother-substance molecules and activated molecules, which, when taken up in circulation restore the pulsations of a non-pulsating heart.

Very probably extraction will also liberate from the skeletal muscles season sensitizers¹⁾, respectively antibodies, whose effect is superadded to that of automatin. The potassium that was also extracted, thanks to the absorption-process, caused no disturbance (at most 5 mgrms of K. per L. of circulating fluid).

The next thing we shall have to try is to follow the automatin on its way from the skeletal muscles via the blood to the heart.

¹⁾ H. ZWAARDEMAKER, K. Akad. van Wetenschappen, Amsterdam, Verslagen van 25 Sept. 1920 en 29 Jan. 1921.

ERRATUM

Page 382, line 5 from top: for 18 read 28.

